

Constants.java (~/Arbeitsfläche/Folien/recDes) - gedit

```
File Edit View Search Tools Documents Help
Open Save Undo Copy Paste Find
Constants.java *
package recDes;
public interface Constants {
    public static final int NULL = -1;
    public static final int PLUS = 0;
    public static final int MINUS = 1;
    public static final int MUL = 2;
    public static final int DIV = 3;
    public static final int LPAR = 4;
    public static final int RPAR = 5;
    public static final int NUMBER = 6;
}
```

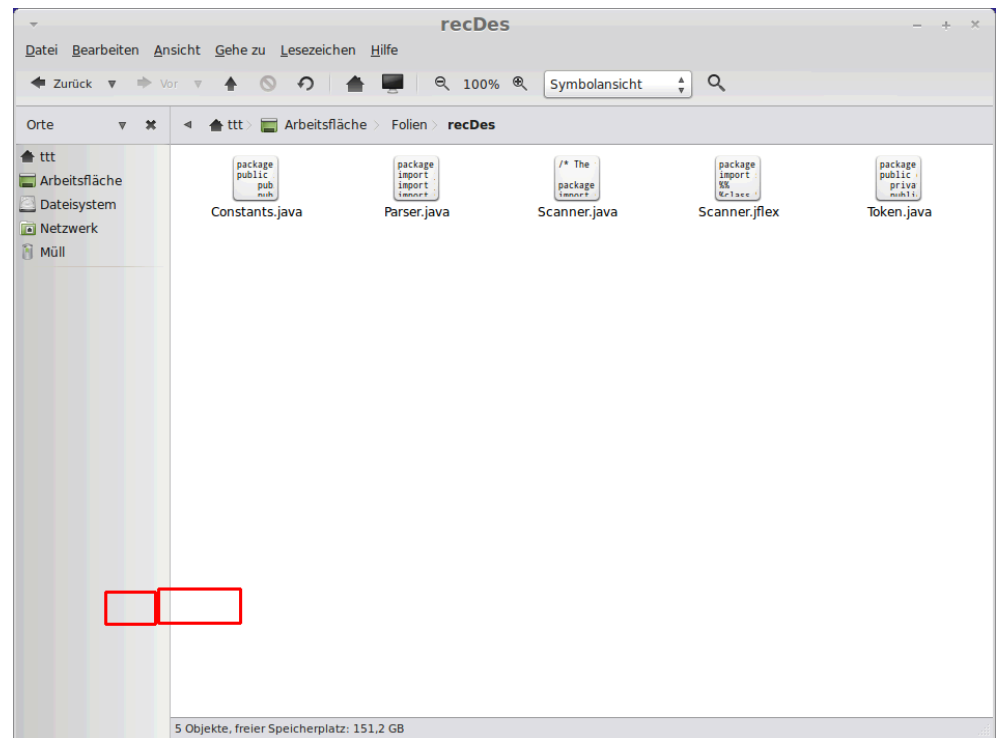
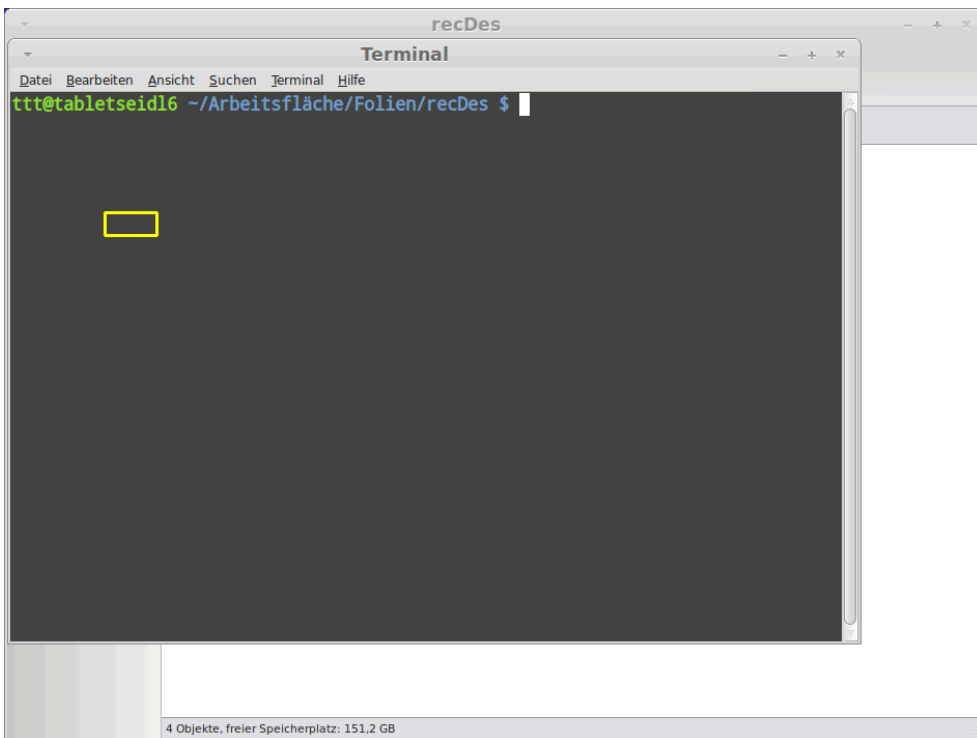
Loading file '/home/ttt/Arbeitsfläche/Folien/recD... Java Tab Width: 8 Ln 1, Col 1 INS

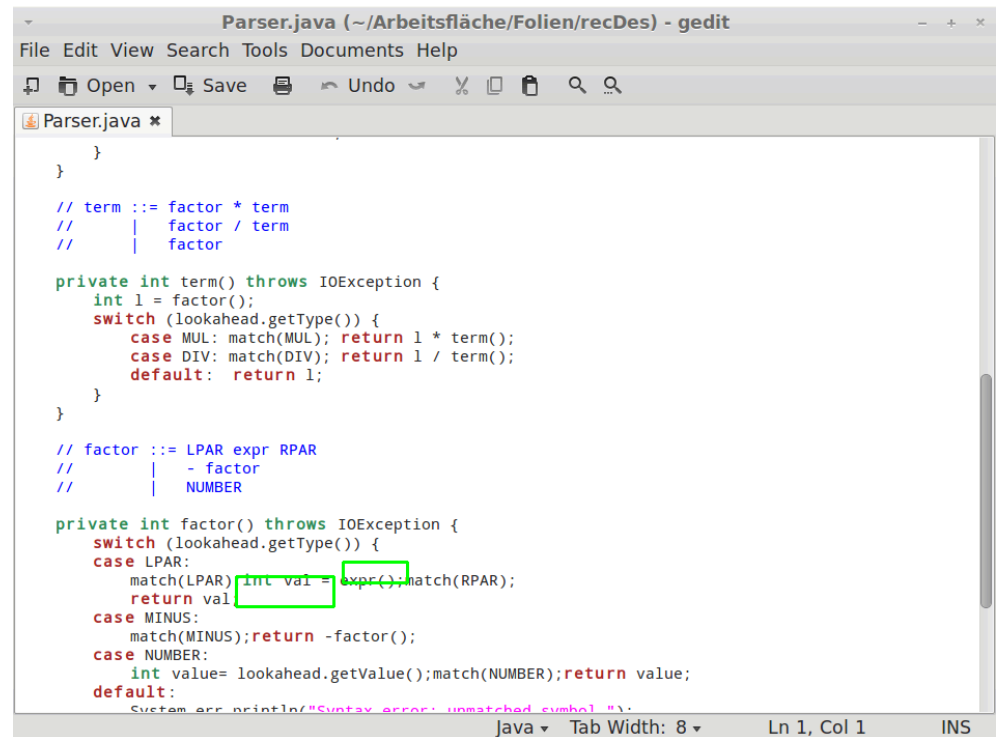
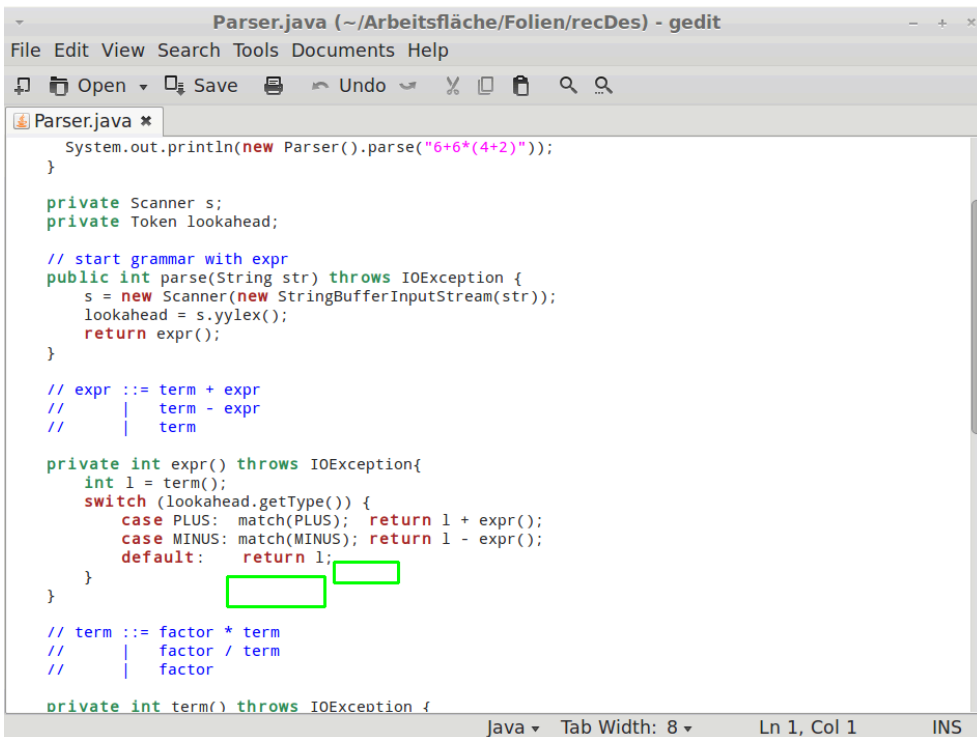
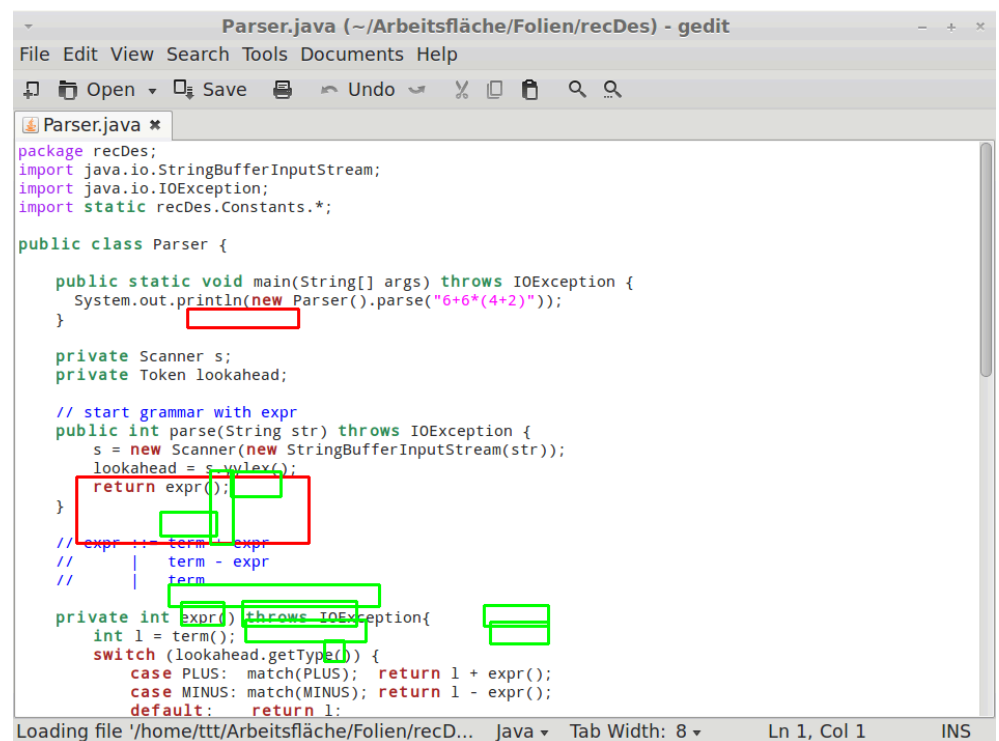
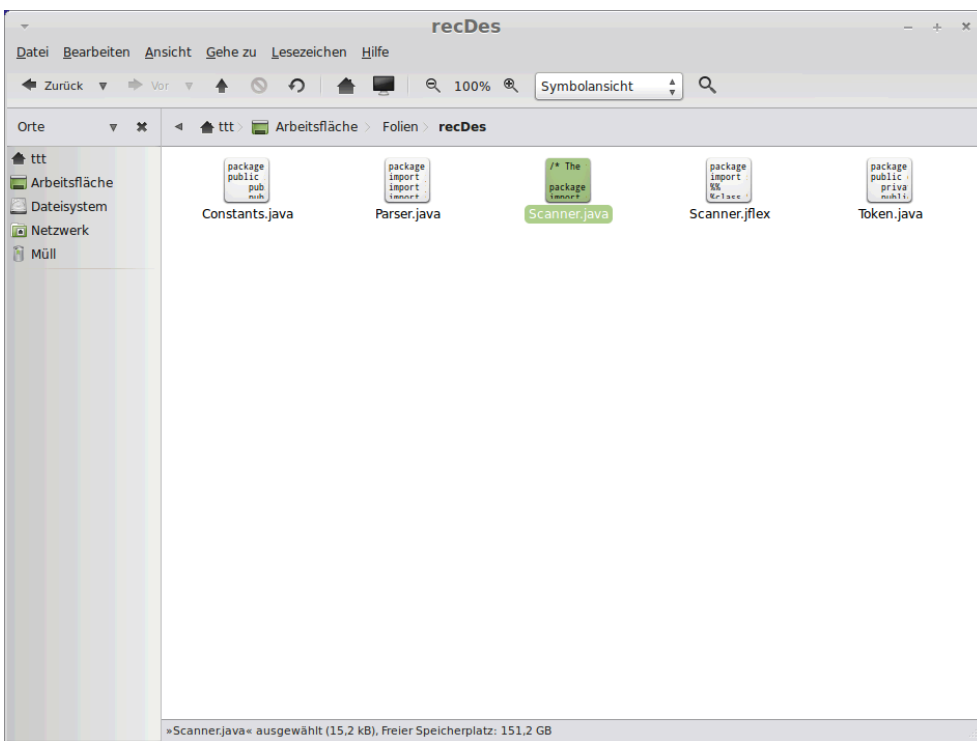
Token.java (~/Arbeitsfläche/Folien/recDes) - gedit

```
File Edit View Search Tools Documents Help
Open Save Undo Copy Paste Find
Token.java *
package recDes;
public class Token {
    private int type;
    public int getType() { return type; }
    private int value;
    public int getValue(){ return value; }

    public Token(int type,int value){
        this.type=type;
        this.value=value;
    }
    public Token(int type){ this(type,0); }
}
```

Loading file '/home/ttt/Arbeitsfläche/Folien/recD... Java Tab Width: 8 Ln 1, Col 1 INS





```
Parser.java (~/Arbeitsfläche/Folien/recDes) - gedit
File Edit View Search Tools Documents Help
Parser.java *
}
// factor ::= LPAR expr RPAR
//         | - factor
//         | NUMBER
private int factor() throws IOException {
    switch (lookahead.getType()) {
        case LPAR:
            match(LPAR);int val = expr();match(RPAR);
            return val;
        case MINUS:
            match(MINUS);return -factor();
        case NUMBER:
            int value= lookahead.getValue();match(NUMBER);return value;
        default:
            System.err.println("Syntax error: unmatched symbol ");
            return 0;
    }
}
private void match(int type) throws IOException {
    if (lookahead.getType() == type)
        lookahead = s.yylex();
    else {
        System.err.println("Syntax error: unexpected Symbol ");
        System.exit(-1);
    }
}
}
Java Tab Width: 8 Ln 9, Col 54 INS
```

```
Parser.java (~/Arbeitsfläche/Folien/recDes) - gedit
File Edit View Search Tools Documents Help
Parser.java *
        case DIV: match(DIV); return 1 / term();
        default: return 1;
    }
}
// factor ::= LPAR expr RPAR
//         | - factor
//         | NUMBER
private int factor() throws IOException {
    switch (lookahead.getType()) {
        case LPAR:
            match(LPAR);int val = expr();match(RPAR);
            return val;
        case MINUS:
            match(MINUS);return -factor();
        case NUMBER:
            int value= lookahead.getValue();match(NUMBER);return value;
        default:
            System.err.println("Syntax error: unmatched symbol ");
            return 0;
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private void match(int type) throws IOException {
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    else {
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        System.exit(-1);
    }
}
}
Java Tab Width: 8 Ln 59, Col 20 INS
```

```
Parser.java (~/Arbeitsfläche/Folien/recDes) - gedit
File Edit View Search Tools Documents Help
Parser.java *
import java.io.StringBufferInputStream;
import java.io.IOException;
import static recDes.Constants.*;
public class Parser {
    public static void main(String[] args) throws IOException {
        System.out.println(new Parser().parse("6+6*(4+2)"));
    }
    private Scanner s;
    private Token lookahead;
    // start grammar with expr
    public int parse(String str) throws IOException {
        s = new Scanner(new StringBufferInputStream(str));
        lookahead = s.yylex();
        return expr();
    }
    // expr ::= term + expr
    //         | term - expr
    //         | term
    private int expr() throws IOException{
        int l = term();
        switch (lookahead.getType()) {
            case PLUS: match(PLUS); return l + expr();
            case MINUS: match(MINUS); return l - expr();
            default: return l;
        }
    }
}
Java Tab Width: 8 Ln 42, Col 45 INS
```

```
recDes
Terminal
Datei Bearbeiten Ansicht Suchen Terminal Hilfe
ttd@tabletseid16 ~/Arbeitsfläche/Folien/recDes $ ..
..: command not found
ttd@tabletseid16 ~/Arbeitsfläche/Folien/recDes $ cd ..
ttd@tabletseid16 ~/Arbeitsfläche/Folien $ javac recDes/*.java
Note: recDes/Parser.java uses or overrides a deprecated API.
Note: Recompile with -Xlint:deprecation for details.
ttd@tabletseid16 ~/Arbeitsfläche/Folien $ java recDes.Parser
Exception in thread "main" java.lang.UnsupportedClassVersionError: recDes/Parser
: Unsupported major.minor version 51.0
    at java.lang.ClassLoader.defineClass1(Native Method)
    at java.lang.ClassLoader.defineClass(ClassLoader.java:643)
    at java.security.SecureClassLoader.defineClass(SecureClassLoader.java:14
2)
    at java.net.URLClassLoader.defineClass(URLClassLoader.java:277)
    at java.net.URLClassLoader.access$000(URLClassLoader.java:73)
    at java.net.URLClassLoader$1.run(URLClassLoader.java:212)
    at java.security.AccessController.doPrivileged(Native Method)
    at java.net.URLClassLoader.findClass(URLClassLoader.java:205)
    at java.lang.ClassLoader.loadClass(ClassLoader.java:323)
    at sun.misc.Launcher$AppClassLoader.loadClass(Launcher.java:294)
    at java.lang.ClassLoader.loadClass(ClassLoader.java:268)
Could not find the main class: recDes.Parser. Program will exit.
ttd@tabletseid16 ~/Arbeitsfläche/Folien $
5 Objekte, freier Speicherplatz: 151.1 GB
```

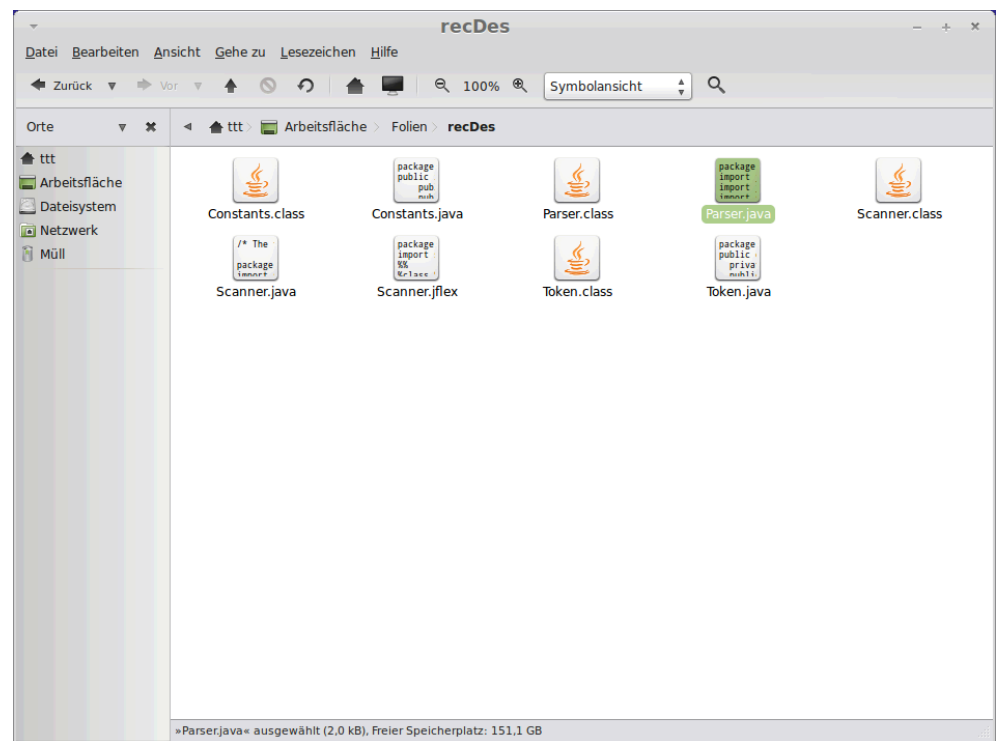
```

recDes
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Datei Bearbeiten Ansicht Suchen Terminal Hilfe
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at java.lang.ClassLoader.loadClass(ClassLoader.java:268)
Could not find the main class: recDes.Parser. Program will exit.
ttt@tabletseidl6 ~/Arbeitsfläche/Folien $ sudo update-alternatives --config java
[sudo] password for ttt:
Es gibt 2 Auswahlmöglichkeiten für die Alternative java (welche /usr/bin/java be
reitstellen).

Auswahl      Pfad                                          Priorität Status
-----
* 0          /usr/lib/jvm/java-6-openjdk-amd64/jre/bin/java 1061    Auto-M
odus
  1          /usr/lib/jvm/java-6-openjdk-amd64/jre/bin/java 1061    manuel
ler Modus
  2          /usr/lib/jvm/java-7-openjdk-amd64/jre/bin/java 1051    manuel
ler Modus

Drücken Sie die Eingabetaste, um die aktuelle Wahl[*] beizubehalten,
oder geben Sie die Auswahlnummer ein:

```



```

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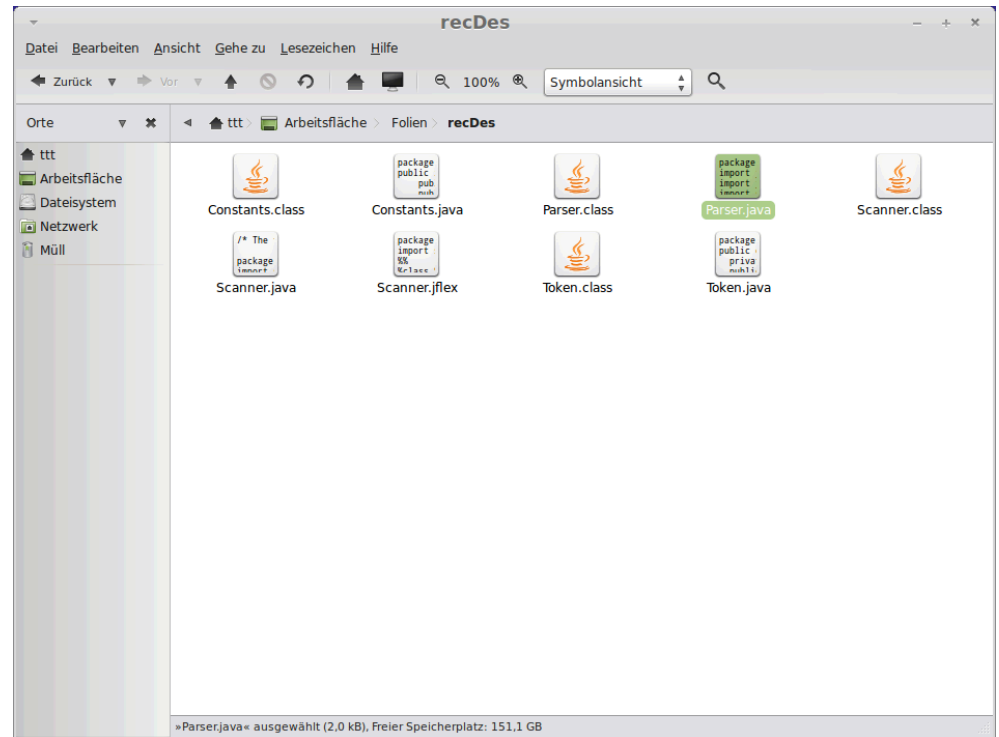
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        return expr();
    }

    // expr ::= term + expr
    //         | term - expr
    //         | term

    private int expr() throws IOException{
        int l = term();
        switch (lookahead.getType()) {
            case PLUS: match(PLUS); return l + expr();
            case MINUS: match(MINUS); return l - expr();
            default: return l;
        }
    }
}

```



Bottom-up Analysis

Attention:

Many grammars are not $LL(k)$!

A reason for that is:

Definition

Grammar G is called **left-recursive**, if

$$A \rightarrow^+ A\beta \quad \text{for an } A \in N, \beta \in (T \cup N)^*$$

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Bottom-up Analysis

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A reason for that is:

Definition

Grammar G is called **left-recursive** if

$$A \rightarrow^+ A\beta \quad \text{for an } A \in N, \beta \in (T \cup N)^*$$

Example:

$$\begin{array}{l|l} E \rightarrow E+T & T \\ T \rightarrow T*F & F \\ F \rightarrow (E) & \text{name} \quad | \quad \text{int} \end{array}$$

... is left-recursive

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Bottom-up Analysis

Theorem:

Let a grammar G be reduced and **left-recursive**, then G is not $LL(k)$ for any k .

Proof:

Let $A \rightarrow A\beta | \alpha \in P$
and A be reachable from S

Assumption: G is $LL(k)$

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Bottom-up Analysis

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$$\Rightarrow \text{First}_k(\alpha\beta^n\gamma) \cap \text{First}_k(\alpha\beta^{n+1}\gamma) = \emptyset$$

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Theorem:

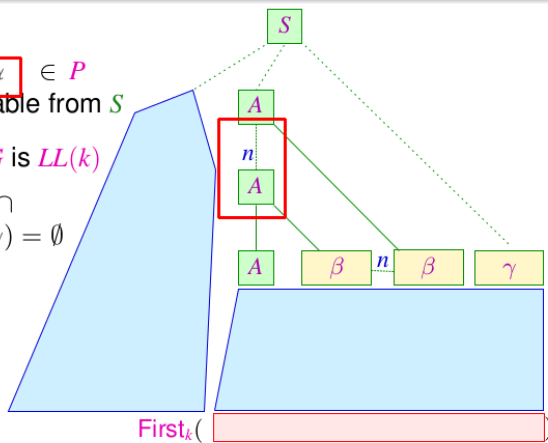
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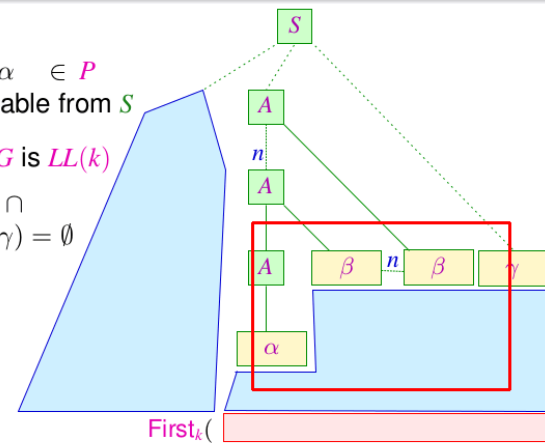
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Case 1: $\beta \rightarrow^* \epsilon$ — Contradiction !!!

Case 2: $\beta \rightarrow^* w \neq \epsilon \implies \text{First}_k(\alpha\beta^k\gamma) \cap \text{First}_k(\alpha\beta^{k+1}\gamma) \neq \emptyset$

Bottom-up Analysis

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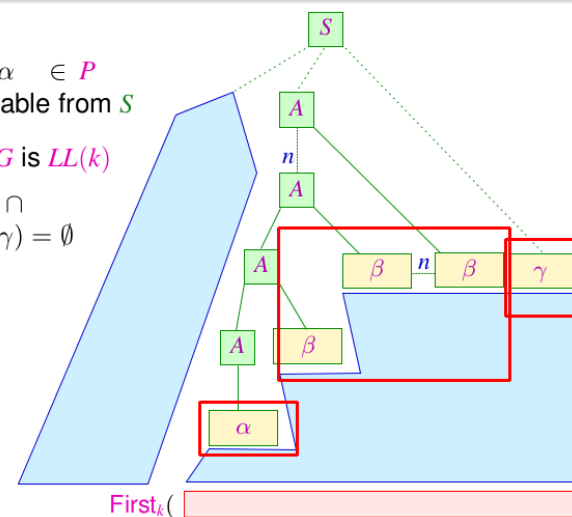
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Shift-Reduce Parser



Donald Knuth

Idea:

We *delay* the decision whether to reduce until we know, whether the input matches the right-hand side of a rule!

Construction: Shift-Reduce parser M_G^R

- The input is shifted successively to the pushdown.
- Is there a complete right-hand side (a handle) atop the pushdown, it is replaced (reduced) by the corresponding left-hand side

Shift-Reduce Parser

Example:

$S \rightarrow AB$
 $A \rightarrow a$
 $B \rightarrow b$

The pushdown automaton:

States: $q_0, f, a, b, A, B, S;$
Start state: q_0
End state: f

q_0	a	$q_0 a$
a	ϵ	A
A	b	Ab
b	ϵ	B
AB	ϵ	S
$q_0 S$	ϵ	f

Shift-Reduce Parser

Construction:

In general, we create an automaton $M_G^R = (Q, T, \delta, q_0, F)$ with:

- $Q = T \cup N \cup \{q_0, f\}$ (q_0, f fresh);
- $F = \{f\}$;
- Transitions:

$$\delta = \left\{ \begin{array}{l} \{(q, x, qx) \mid q \in Q, x \in T\} \cup \quad // \text{ Shift-transitions} \\ \{(q\alpha, \epsilon, qA) \mid q \in Q, A \rightarrow \alpha \in P\} \cup \quad // \text{ Reduce-transitions} \\ \{(q_0 S, \epsilon, f)\} \quad // \text{ finish} \end{array} \right.$$

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Shift-Reduce Parser

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Example-computation:

$$\begin{array}{l} (q_0, ab) \vdash (q_0 a, b) \vdash (q_0 A, b) \\ \vdash (q_0 A b, \epsilon) \vdash (q_0 AB, \epsilon) \\ \vdash (q_0 S, \epsilon) \vdash (f, \epsilon) \end{array}$$

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Shift-Reduce Parser

Observation:

- The sequence of reductions corresponds to a **reverse rightmost-derivation** for the input
- To prove correctness, we have to prove:

$$(\epsilon, w) \vdash^* (A, \epsilon) \quad \text{iff} \quad A \rightarrow^* w$$

- The shift-reduce pushdown automaton M_G^R is in general also **non-deterministic**
- For a deterministic parsing-algorithm, we have to identify computation-states for reduction

⇒ LR-Parsing

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Shift-Reduce Parser

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In general, we create an automaton $M_G^R = (Q, T, \delta, q_0, F)$ with:

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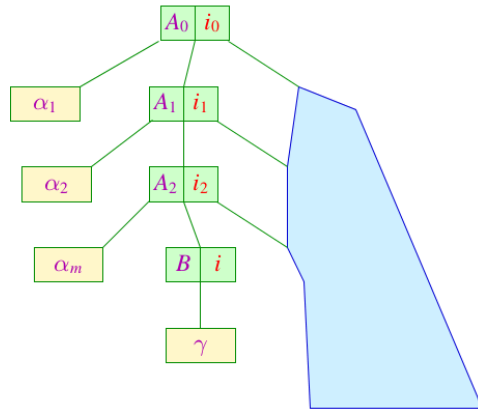
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Bottom-up Analysis

$\alpha\gamma$ is viable for $[B \rightarrow \gamma \bullet]$ iff $S \xrightarrow{*}_R \alpha B v$

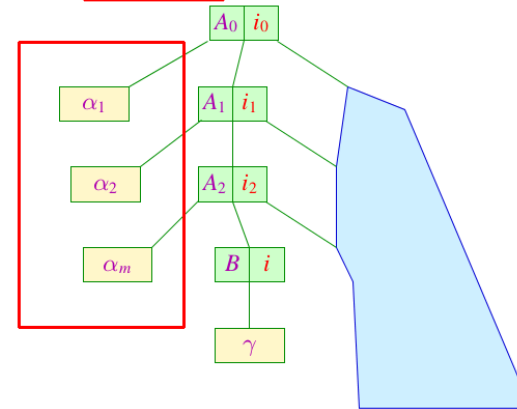


... with $\alpha = \alpha_1 \dots \alpha_m$

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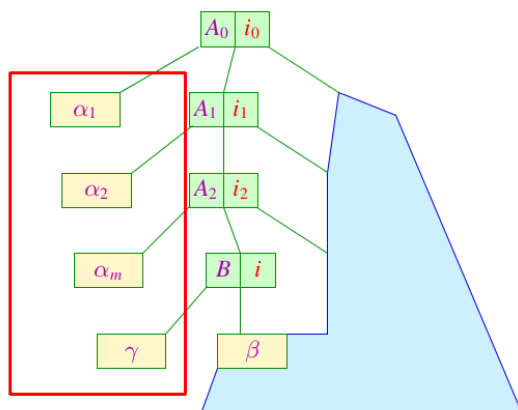
... with $\alpha = \alpha_1 \dots \alpha_m$

Conversely, for an arbitrary valid word α' we can determine the set of all later on possibly matching rules ...

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Bottom-up Analysis

The item $[B \rightarrow \gamma \bullet \beta]$ is called **admissible** for α' iff $S \xrightarrow{*}_R \alpha B v$ with $\alpha' = \alpha \gamma$:



... with $\alpha = \alpha_1 \dots \alpha_m$

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Characteristic Automaton

Observation:

The set of viable prefixes from $(N \cup T)^*$ for (admissible) items can be computed from the content of the **shift-reduce parser's** pushdown with the help of a finite automaton:

States: Items

Start state: $[S' \rightarrow \bullet S]$

Final states: $\{[B \rightarrow \gamma \bullet] \mid B \rightarrow \gamma \in P\}$

Transitions:

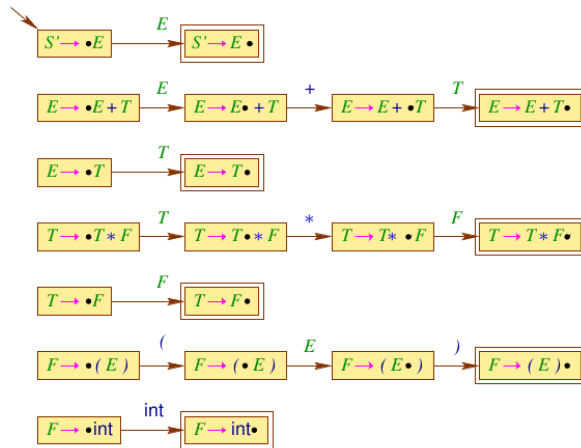
- (1) $([A \rightarrow \alpha \bullet X \beta], [X], [A \rightarrow \alpha X \bullet \beta]), [X \in (N \cup T)], A \rightarrow \alpha X \beta \in P;$
- (2) $([A \rightarrow \alpha \bullet B \beta], \epsilon, [B \rightarrow \bullet \gamma]), A \rightarrow \alpha B \beta, B \rightarrow \gamma \in P;$

The automaton $c(G)$ is called **characteristic automaton** for G .

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Characteristic Automaton

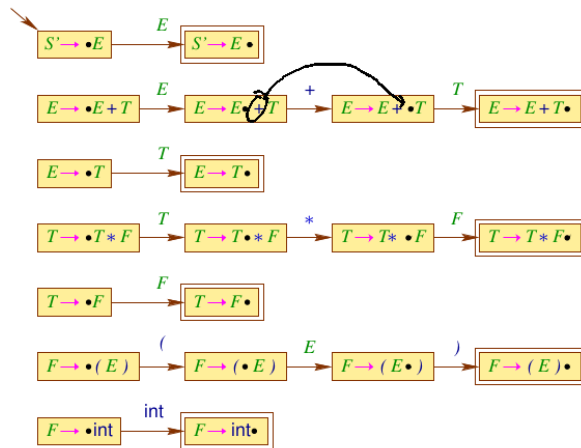
for example:

$$\begin{array}{l|l} E & \rightarrow E+T & | & T \\ T & \rightarrow T*F & | & F \\ F & \rightarrow (E) & | & \text{int} \end{array}$$


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Characteristic Automaton

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Final states: $\{[B \rightarrow \gamma \bullet] \mid B \rightarrow \gamma \in P\}$

Transitions:

- (1) $([A \rightarrow \alpha \bullet X \beta], X, [A \rightarrow \alpha X \bullet \beta])$, $X \in (N \cup T), A \rightarrow \alpha X \beta \in P$;
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Characteristic Automaton

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Start state: $[S' \rightarrow \bullet S]$

Final states: $\{[B \rightarrow \gamma \bullet] \mid B \rightarrow \gamma \in P\}$

Transitions:

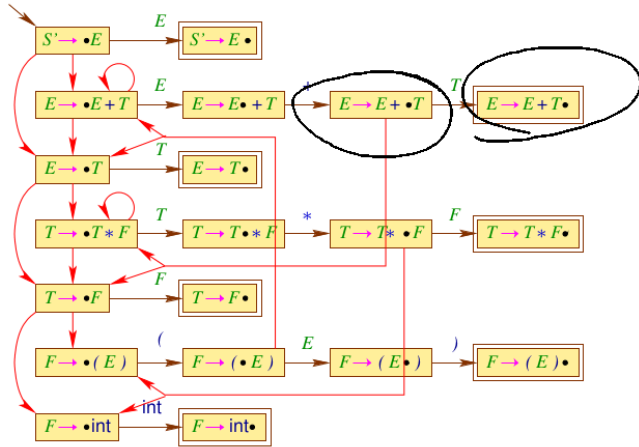
- (1) $([A \rightarrow \alpha \bullet X \beta], X, [A \rightarrow \alpha X \bullet \beta])$, $X \in (N \cup T), A \rightarrow \alpha X \beta \in P$;
- (2) $([A \rightarrow \alpha \bullet B \beta], \epsilon, [B \rightarrow \bullet \gamma])$, $A \rightarrow \alpha B \beta, B \rightarrow \gamma \in P$;

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Characteristic Automaton

for example:

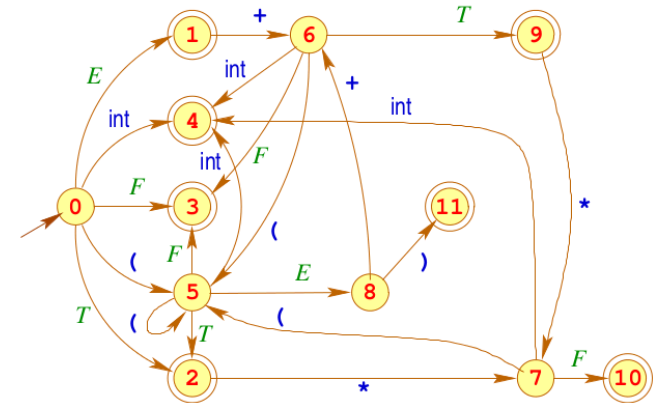
$$\begin{array}{l} E \rightarrow E+T \quad | \quad T \\ T \rightarrow T*F \quad | \quad F \\ F \rightarrow (E) \quad | \quad \text{int} \end{array}$$


Canonical LR(0)-Automaton

The canonical LR(0)-automaton $LR(G)$ is created from $c(G)$ by:

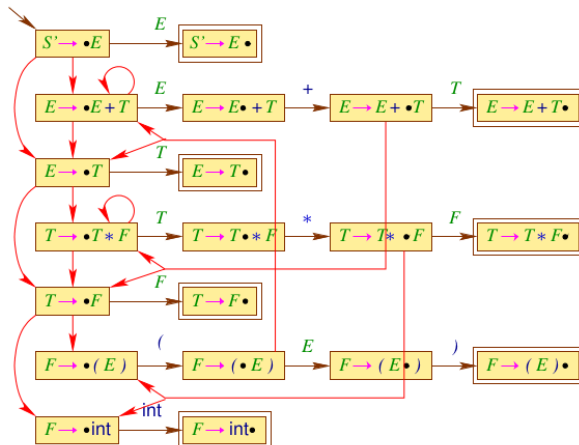
- 1 performing arbitrarily many ϵ -transitions after every consuming transition
- 2 performing the powerset construction

... for example:



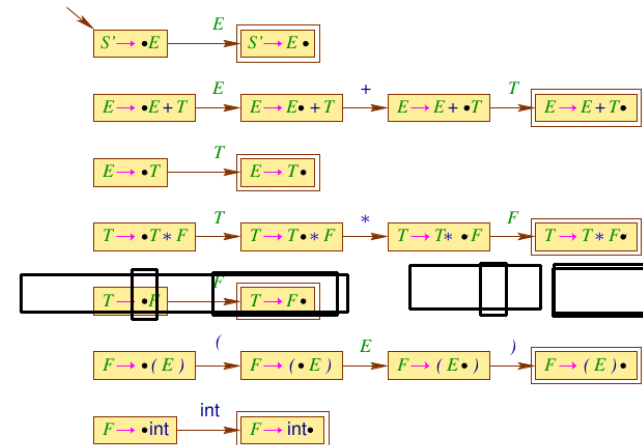
Characteristic Automaton

for example:

$$\begin{array}{l} E \rightarrow E+T \quad | \quad T \\ T \rightarrow T*F \quad | \quad F \\ F \rightarrow (E) \quad | \quad \text{int} \end{array}$$


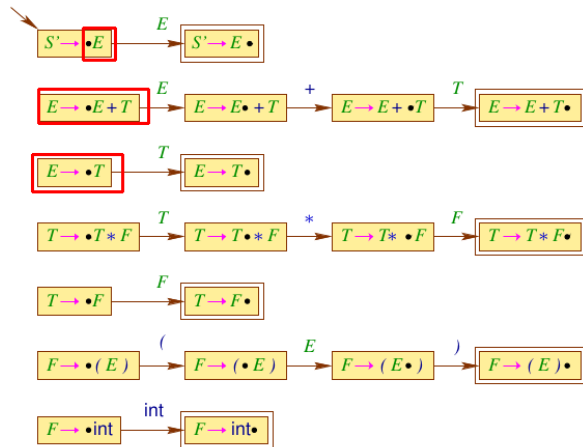
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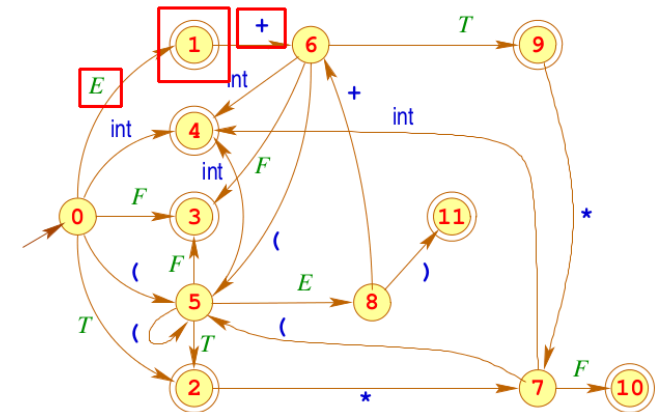
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Canonical LR(0)-Automaton

The canonical LR(0)-automaton $LR(G)$ is created from $c(G)$ by:

- performing arbitrarily many ϵ -transitions after every consuming transition
- performing the powerset construction

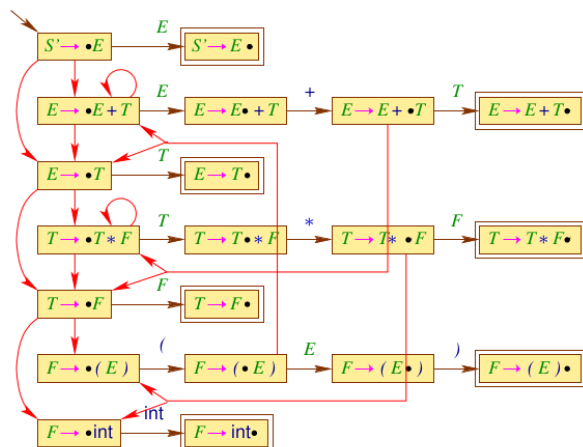
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Characteristic Automaton

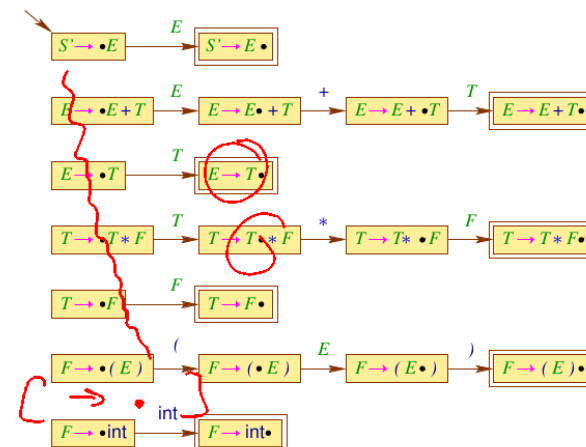
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Characteristic Automaton

$[4] [7] [8]$
nt

Observation:

The set of viable prefixes from $(N \cup T)^*$ for (admissible) items can be computed from the content of the shift-reduce parser's pushdown with the help of a finite automaton:

States: Items

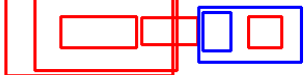
Start state: $[S' \rightarrow \bullet S]$

Final states: $\{[B \rightarrow \gamma \bullet] \mid B \rightarrow \gamma \in P\}$

Transitions:

- (1) $([A \rightarrow \alpha \bullet X \beta], [A \rightarrow \alpha X \bullet \beta]), X \in (N \cup T), A \rightarrow \alpha X \beta \in P;$
- (2) $([A \rightarrow \alpha \bullet B \beta], [B \rightarrow \bullet \gamma]), A \rightarrow \alpha B \beta, B \rightarrow \gamma \in P;$

The automaton $c(G)$ is called characteristic automaton for G .



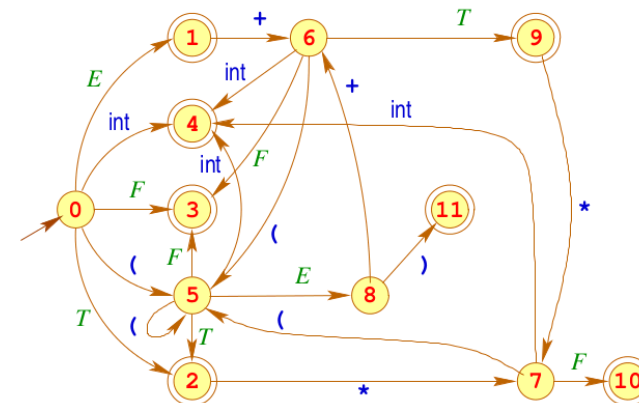
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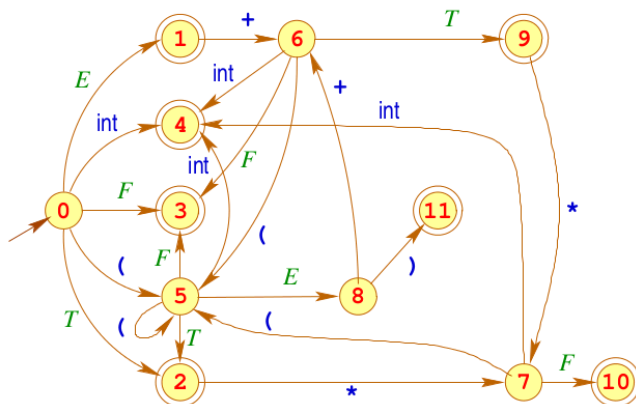
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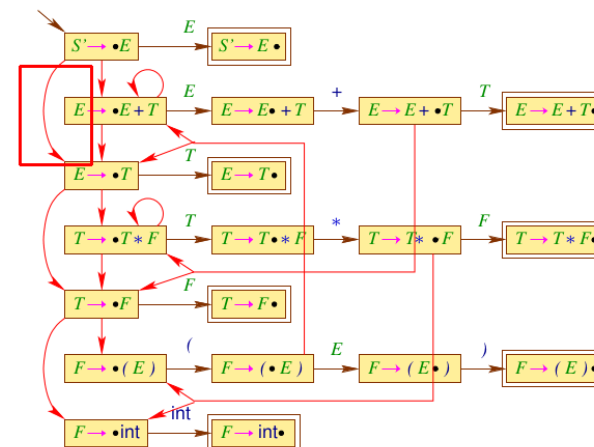


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Characteristic Automaton

for example:

$E \rightarrow E+T \mid T$
 $T \rightarrow T*F \mid F$
 $F \rightarrow (E) \mid \text{int}$



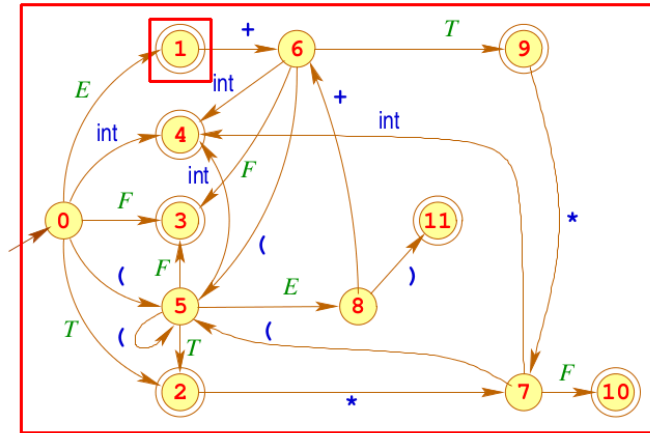
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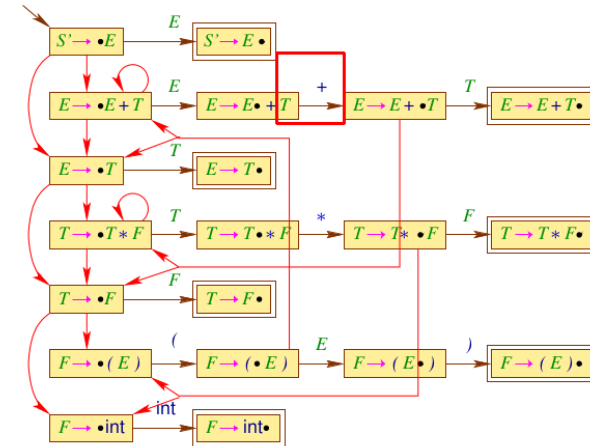
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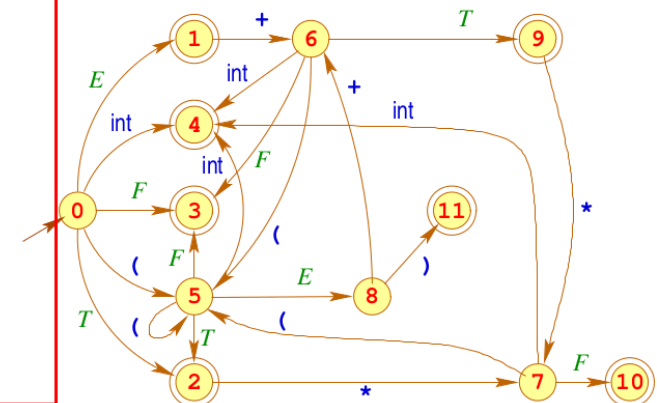
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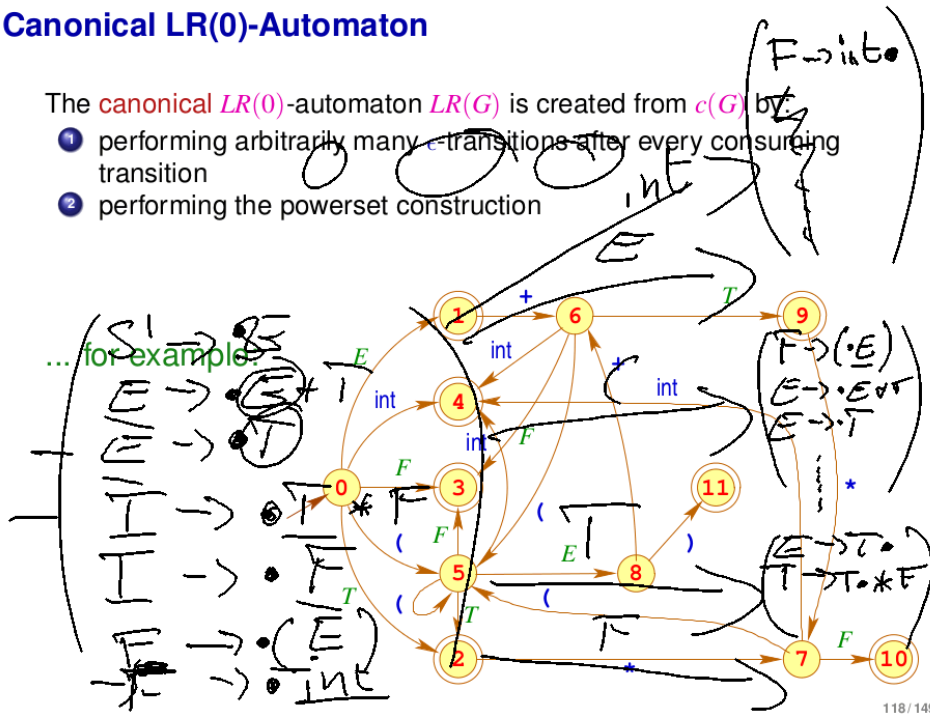
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Canonical LR(0)-Automaton

Example:

$E \rightarrow E + T \mid T$
 $T \rightarrow T * F \mid F$
 $F \rightarrow (E) \mid \text{int}$

Therefore we determine:

$$\begin{aligned}
 q_0 &= \{ [S' \rightarrow \bullet E], [E \rightarrow \bullet E + T], [E \rightarrow \bullet T], [T \rightarrow \bullet T * F], [T \rightarrow \bullet F], [F \rightarrow \bullet (E)], [F \rightarrow \bullet \text{int}] \} \\
 q_1 &= \delta(q_0, E) = \{ [S' \rightarrow E \bullet], [E \rightarrow E \bullet + T] \} \\
 q_2 &= \delta(q_0, T) = \{ [E \rightarrow T \bullet], [T \rightarrow T \bullet * F] \} \\
 q_3 &= \delta(q_0, F) = \{ [T \rightarrow F \bullet] \} \\
 q_4 &= \delta(q_0, \text{int}) = \{ [F \rightarrow \text{int} \bullet] \}
 \end{aligned}$$

Canonical LR(0)-Automaton

$$\begin{aligned}
 q_5 &= \delta(q_0, () = \{ [F \rightarrow (\bullet E)], [E \rightarrow E \bullet + T], [E \rightarrow \bullet T], [T \rightarrow \bullet T * F], [T \rightarrow \bullet F], [F \rightarrow \bullet (E)], [F \rightarrow \bullet \text{int}] \} \\
 q_6 &= \delta(q_1, +) = \{ [E \rightarrow E + \bullet T], [T \rightarrow \bullet T * F], [T \rightarrow \bullet F], [F \rightarrow \bullet (E)], [F \rightarrow \bullet \text{int}] \} \\
 q_7 &= \delta(q_2, *) = \{ [T \rightarrow T * \bullet F], [F \rightarrow \bullet (E)], [F \rightarrow \bullet \text{int}] \} \\
 q_8 &= \delta(q_5, E) = \{ [F \rightarrow (E \bullet)], [E \rightarrow E \bullet + T] \} \\
 q_9 &= \delta(q_6, T) = \{ [E \rightarrow E + T \bullet], [T \rightarrow T \bullet * F] \} \\
 q_{10} &= \delta(q_7, F) = \{ [T \rightarrow T * F \bullet] \} \\
 q_{11} &= \delta(q_8,) = \{ [F \rightarrow (E) \bullet] \}
 \end{aligned}$$

Canonical LR(0)-Automaton

Observation:

The canonical LR(0)-automaton can be created directly from the grammar.

Therefore we need a helper function δ_ϵ^* (ϵ -closure)

$$\delta_\epsilon^*(q) = q \cup \{ [B \rightarrow \bullet \gamma] \mid \exists [A \rightarrow \alpha \bullet B' \beta'] \in q, \beta' \in (N \cup T)^* : B' \rightarrow^* B \beta \}$$

We define:

States: Sets of items;

Start state: $\delta_\epsilon^* \{ [S' \rightarrow \bullet S] \}$

Final states: $\{ q \mid \exists A \rightarrow \alpha \in P : [A \rightarrow \alpha \bullet] \in q \}$

Transitions: $\delta(q, X) = \delta_\epsilon^* \{ [A \rightarrow \alpha X \bullet \beta] \mid [A \rightarrow \alpha \bullet X \beta] \in q \}$

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LR(0)-Parser

... for example:

$$\begin{aligned} q_1 &= \{ [S' \rightarrow E \bullet], [E \rightarrow E \bullet + T] \} \\ q_2 &= \{ [E \rightarrow T \bullet], [T \rightarrow T \bullet * F] \} \\ q_3 &= \{ [T \rightarrow F \bullet] \} \\ q_4 &= \{ [F \rightarrow \text{int} \bullet] \} \\ q_9 &= \{ [E \rightarrow E + T \bullet], [T \rightarrow T \bullet * F] \} \\ q_{10} &= \{ [T \rightarrow T * F \bullet] \} \\ q_{11} &= \{ [F \rightarrow (E) \bullet] \} \end{aligned}$$

The final states q_1, q_2, q_9 contain more than one admissible item
 \Rightarrow non deterministic!

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LR(0)-Parser

Idea for a parser:

- The parser manages a viable prefix $\alpha = X_1 \dots X_m$ on the pushdown and uses $LR(G)$, to identify reduction spots.
- It can reduce with $A \rightarrow \gamma$, if $[A \rightarrow \gamma \bullet]$ is admissible for α

Optimization:

We push the states instead of the X_i in order not to process the pushdown's content with the automaton anew all the time.

Reduction with $A \rightarrow \gamma$ leads to popping the uppermost $|\gamma|$ states and continue with the state on top of the stack and input A .

Attention:

This parser is only deterministic, if each final state of the canonical LR(0)-automaton is conflict free.

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LR(0)-Parser

The construction of the LR(0)-parser:

States: $Q \cup \{f\}$ (f fresh)

Start state: q_0

Final state: f

Transitions:

Shift: $(p, a, p q)$ if $q = \delta(p, a) \neq \emptyset$

Reduce: $(p q_1 \dots q_m, \epsilon, p q)$ if $[A \rightarrow X_1 \dots X_m \bullet] \in q_m$,
 $q = \delta(p, A)$

Finish: $(q_0 p, \epsilon, f)$ if $[S' \rightarrow S \bullet] \in p$

with $LR(G) = (Q, T, \delta, q_0, F)$.

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