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Title: FDS (26.07.2019)

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Duration: 97:40 min

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20 Amortized Complexity

21 Skew Heap

22 Splay Tree

23 Pairing Heap

24 More Verified Data Structures and Algorithms (in Isabelle/HOL)

Implementation type

datatype $'a\ heap = Empty \mid Hp\ 'a\ ('a\ heap\ list)$

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Heap invariant:

$pheap\ Empty = True$
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Also: *Empty* must only occur at the root

insert

$insert\ x\ h = merge\ (Hp\ x\ [])\ h$

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$merge\ (Hp\ x\ lx =: hx)\ (Hp\ y\ ly =: hy) =$
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Like function *link* for binomial heaps

del_min

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$pass_2\ [] = Empty$
$pass_2\ (h\ \#\ hs) = merge\ h\ (pass_2\ hs)$

Fusing $pass_2 \circ pass_1$

$merge\_pairs\ [] = Empty$
$merge\_pairs\ [h] = h$
$merge\_pairs\ (h_1\ \#\ h_2\ \#\ hs) =$
$merge\ (merge\ h_1\ h_2)\ (merge\_pairs\ hs)$

Functional correctness proofs

Straightforward

Analysis

Analysis easier (more uniform) if a pairing heap is viewed as a binary tree:

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Potential function same as for splay trees

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Analysis of *splay*

Theorem

$$[\![bst\ t;\ \langle l,\ a,\ r\rangle \in subtrees\ t]\!]$$
$$\Longrightarrow a\_splay\ a\ t \leq 3 * (\varphi\ t - \varphi\ \langle l,\ a,\ r\rangle) + 1$$

Corollary

$$[\![bst\ t;\ a \in set\_tree\ t]\!]$$
$$\Longrightarrow a\_splay\ a\ t \leq 3 * (\varphi\ t - 1) + 1$$

Verified:

The functions $insert$, del_min and $merge$ all have $O(\log_2 n)$ amortized complexity.

These bounds are not tight.

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These bounds are not tight.
Better amortized bounds in the literature:
$insert \in O(1)$, $del\_min \in O(\log_2 n)$, $merge \in O(1)$

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Archive of Formal Proofs

`https://www.isa-afp.org/entries/Amortized_Complexity.shtml`

Sources

The inventors of the pairing heap:

M. Fredman, R. Sedgewick, D. Sleator and R. Tarjan.
The Pairing Heap: A New Form of Self-Adjusting Heap.
Algorithmica, 1986.

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More trees

- Huffman Trees:

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  Huffman 1952 / Blanchette 2008

Graph algorithms

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  Floyd 1962, Warshall 1962 /
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- Strongly Connected Components:
  Tarjan 1972 / Schimpf 2015
  Gabow 2000 / Lammich 2014
- Minimum spanning tree:
  Kruskal 1956, Prim 1957 /
  Guttmann 2018, Lammich *et al.* 2019

Model Checkers

- SPIN-like LTL Model Checker:
  Esparza, Lammich, Neumann, Nipkow, Schimpf,
  Smaus 2013

Dynamic programming

- Start with recursive function

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- Automatic translation to memoized version incl. correctness theorem

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- Start with recursive function
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- Applications
  - Optimal binary search tree
  - Minimum edit distance
  - Bellman-Ford (SSSP)
  - CYK
  - ...

Infrastructure

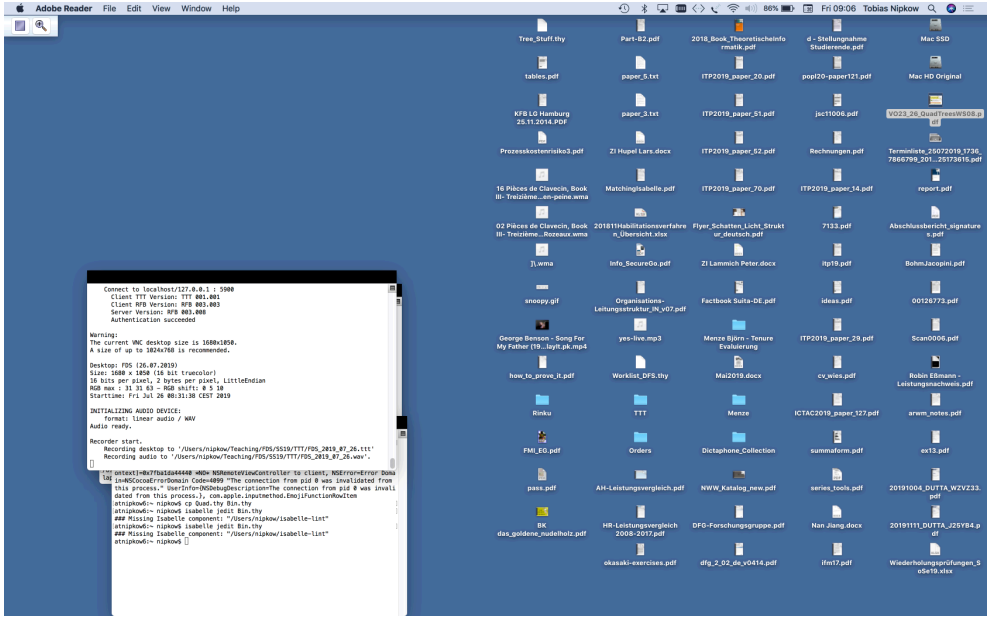
Refinement Frameworks by Lammich:

Infrastructure

Refinement Frameworks by Lammich:

Abstract specification
$\rightsquigarrow$ functional program
$\rightsquigarrow$ imperative program

Mostly in the Archive of Formal Proofs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Factbook Suita-DE.pdf
ideas.pdf
00126773.pdf
George Benson - Song For My Father (19...layit.pk.mp4
yes-live.mp3
Menze Björn - Tenure Evaluierung
ITP2019_paper_29.pdf
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Worklist_DFS.thy
Mai2019.docx
cv_wies.pdf
Robin EBmann - Leistungsnachweis.pdf
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AH-Leistungsvergleich.pdf
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DFG-Forschungsgruppe.pdf
Nan Jiang.docx
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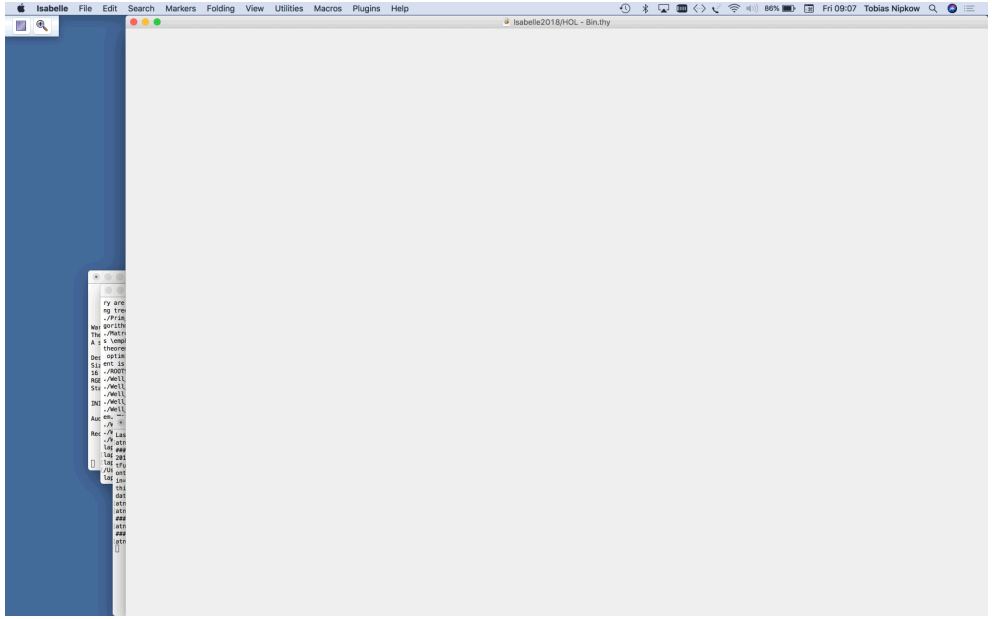
okasaki-exercises.pdf
dfg_2_02_de_v0414.pdf
ifm17.pdf
Wiederholungsprüfungen_SoSe19.xlsx
Connect to localhost/127.0.0.1 : 5900
Client TTT Version: TTT 003.001
Client RFB Version: RFB 003.003
Server Version: RFB 003.008
Authentication succeeded
Warning:
The current VNC desktop size is 1680x1050.
A size of up to 10244788 is recommended.
Desktop: FDS (26.07.2019)
Size: 1680 x 1050 (16 bit truecolor)
16 bits per pixel, 2 bytes per pixel, LittleEndian
RGB max : 31 31 63 - RGB shift: 0 5 10
Starttime: Fri Jul 26 00:31:38 CEST 2019
INITIALIZING AUDIO DEVICE:
format: linear audio / WAV
Audio ready.
Recorder start.
Recording desktop to '/Users/nipkow/Teaching/FDS/SS19/TTT/FDS_2019_07_26.ttt'
Recording audio to '/Users/nipkow/Teaching/FDS/SS19/TTT/FDS_2019_07_26.wav'.
atnipkow6:~ nipkow$ cp Quad.thy Bin.thy
atnipkow6:~ nipkow$ isabelle jedit Bin.thy
Missing Isabelle component: "/Users/nipkow/isabelle-lint"
atnipkow6:~ nipkow$ isabelle jedit Bin.thy
Missing Isabelle component: "/Users/nipkow/isabelle-lint"
atnipkow6:~ nipkow$


Isabelle File Edit Search Markers Folding View Utilities Macros Plugins Help
Fri 09:07 Tobias Nipkow
isabelle2018/HOL - Bin.thy