Script **generated by TTT**

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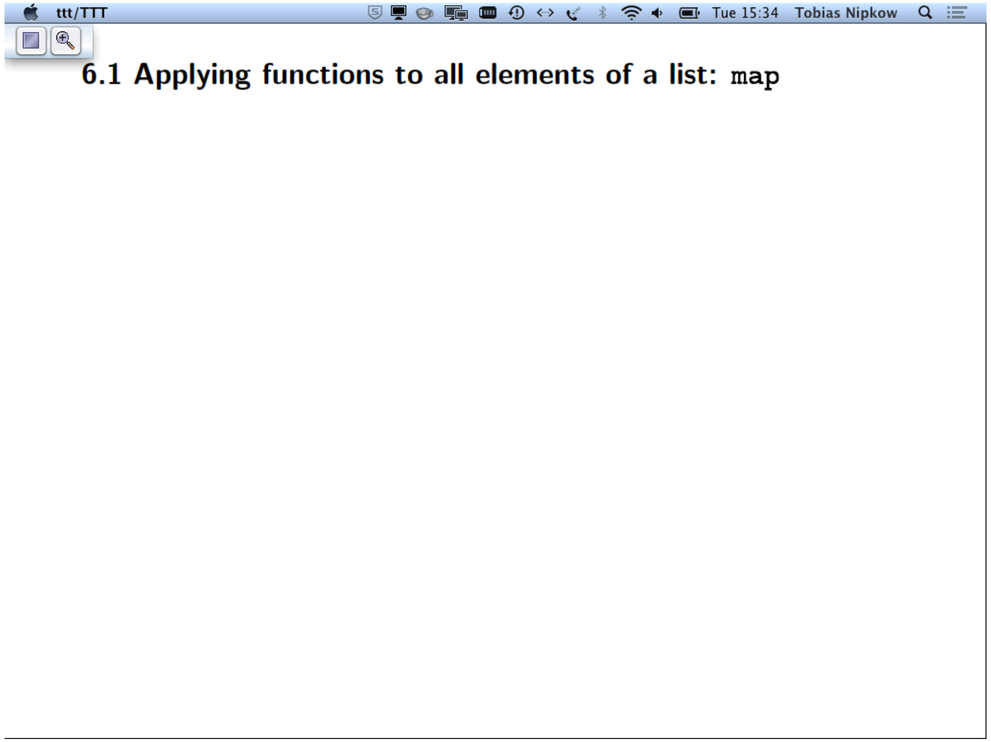
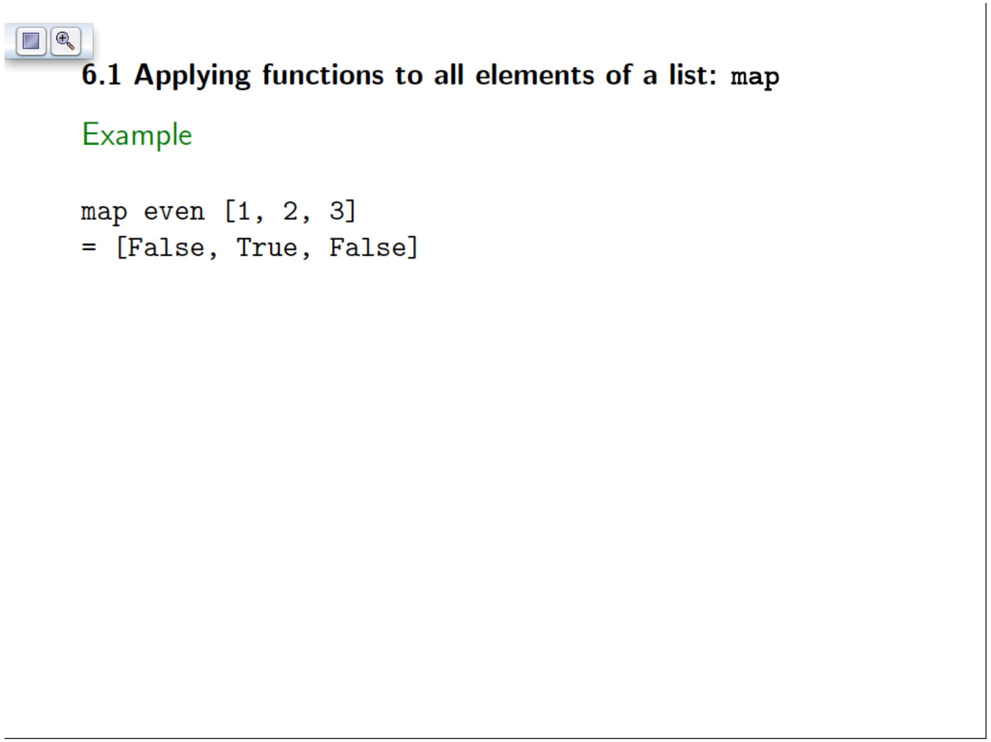


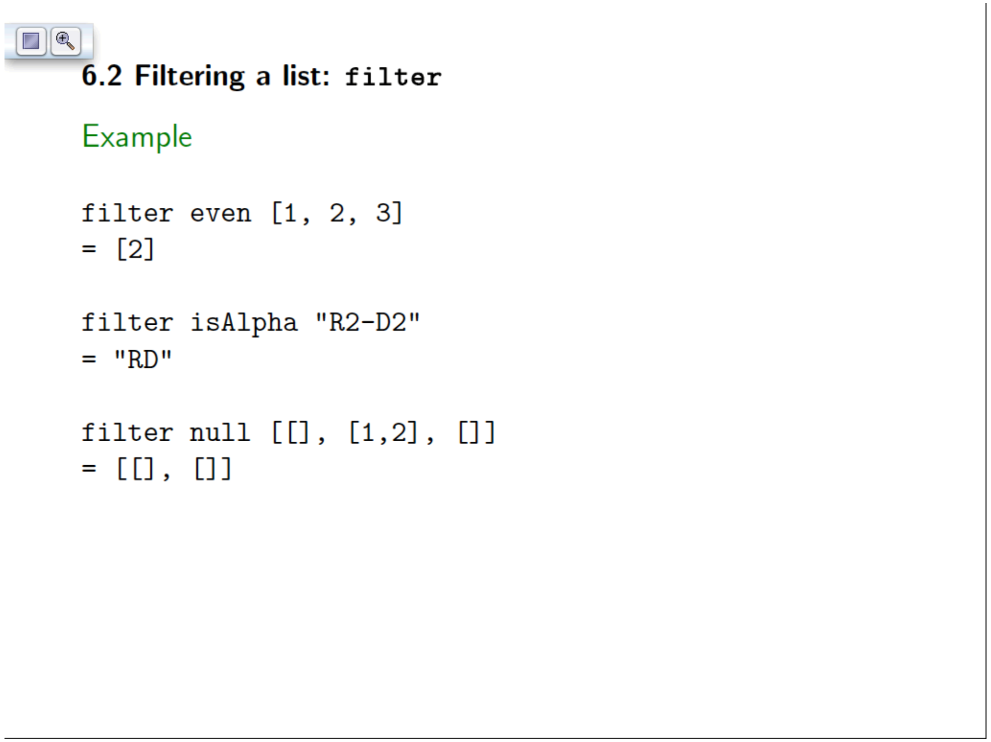
6.1 Applying functions to all elements of a list: `map`



6.1 Applying functions to all elements of a list: `map`

Example

```
map even [1, 2, 3]
= [False, True, False]
```



6.2 Filtering a list: `filter`

Example

```
filter even [1, 2, 3]
= [2]

filter isAlpha "R2-D2"
= "RD"

filter null [[], [1,2], []]
= [[], []]
```

6.3 Combining the elements of a list: `foldr`

Example

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sum []        =  0
sum (x:xs)    =  x + sum xs
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concat (xs:xss)    =  xs ++ concat xss
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$$\mathtt{concat}\ [xs_1,\ \ldots,\ xs_n] = xs_1 \mathbin{+\!\!+} \ldots \mathbin{+\!\!+} xs_n \mathbin{+\!\!+} []$$

foldr

$\texttt{foldr}\ (\oplus)\ z\ [x_1,\ \ldots,\ x_n] = x_1 \oplus \ldots\ \oplus x_n \oplus z$

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Defined in `Prelude`:

```
foldr :: (a -> a -> a) -> a -> [a] -> a
foldr f a []       =  a
foldr f a (x:xs)   =  x ‘f‘ foldr f a xs
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Applications:

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sum xs   =  foldr (+) 0 xs

concat xss =  foldr (++) [] xss
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What is the most general type of `foldr`?

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foldr f a []        =  a
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foldr (+) 0 [1, -2]
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foldr f a (x:xs)    =  x ‘f‘ foldr f a xs

foldr (+) 0 [1, -2]
= foldr (+) 0 (1 : -2 : [])
= 1 + foldr (+) 0 (-2 : [])
= 1 + -2 + (foldr (+) 0 [])
= 1 + -2 + 0
= -1
```

More applications of foldr

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product xs  =  foldr  (*)   1      xs
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product xs  =  foldr  (*)  1     xs
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product xs  =  foldr  (*)  1     xs
and xs      =  foldr  (&&) True  xs
or xs       =  foldr  (||) False xs
inSort xs   =  foldr  ins  []    xs
```

Quiz

What is

`foldr (:) ys xs`

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Example: `foldr (:) ys (1:2:3:[]) = 1:2:3:ys`

`foldr (:) ys xs = xs ++ ys`

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Proof by induction on `xs`. Induction step:
`foldr f a ((x:xs) ++ ys)`

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Therefore, if `g xs = foldr f a xs`,
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Therefore, if `g xs = foldr f a xs`,
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Therefore `sum (xs++ys) = sum xs + sum ys`,
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Usage:

```
squares xs  =  map (\x -> x * x) xs
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Terminology

`(\x ->` e_1`)` e_2

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x: formal parameter
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Example

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List comprehension

Just syntactic sugar for combinations of `map`

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[ f x | x <- xs ]     =     map f xs
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and `concat`

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6.5 Extensionality

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f x y  =  x+y              f x  =  \y -> x+y
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f :: Int -> Int -> Int     f :: Int -> (Int -> Int)
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Both mean the same:

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Both mean the same:

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= a + b                     = (\y -> a + y) b
                            = a + b
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The trick: any function of two arguments
can be viewed as a function of the first argument

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Both mean the same:

```
f a b                       (f a) b
= a + b                     = (\y -> a + y) b
                            = a + b
```

The trick: any function of two arguments
can be viewed as a function of the first argument
that returns a function of the second argument

In general

Every function is a function of one argument
(which may return a function as a result)

$$T_1 \texttt{ -> } T_2 \texttt{ -> } T$$

is just syntactic sugar for

$$T_1 \texttt{ -> } (T_2 \texttt{ -> } T)$$

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$$\underbrace{(f \;\; e_1)}_{::\, T_2 \,\texttt{->}\, T} \;\; e_2$$

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Example

```
f :: Int -> (Int -> Int)
f x y  =  x + y
```

`->` is not associative:

$$T_1 \;\texttt{->}\; (T_2 \;\texttt{->}\; T) \neq (T_1 \;\texttt{->}\; T_2) \;\texttt{->}\; T$$

Example

```
f :: Int -> (Int -> Int)     g :: (Int -> Int) -> Int
f x y  =  x + y              g h  =  h 0 + 1
```

`->` is not associative:

$$T_1 \;\texttt{->}\; (T_2 \;\texttt{->}\; T) \neq (T_1 \;\texttt{->}\; T_2) \;\texttt{->}\; T$$

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f :: Int -> (Int -> Int)      g :: (Int -> Int) -> Int
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Application is not associative:

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Example

`(f 3) 4` ≠ `f (3 4)`

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Example

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f :: Int -> (Int -> Int)      g :: (Int -> Int) -> Int
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```

Application is not associative:

$$(f\;\; e_1)\;\; e_2 \neq f\;\; (e_1\;\; e_2)$$

Example

`(f 3) 4` ≠ `f (3 4)`   `g (id abs)` ≠ `(g id) abs`

Quiz

`head tail xs`

Correct?

Partial application

Every function of n parameters
can be applied to less than n arguments

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Example

```
Instead of    sum xs  =  foldr (+) 0 xs
just define   sum     =  foldr (+) 0
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Example

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In general:

If $f \; \texttt{::} \; T_1 \; \texttt{->} \; \ldots \; \texttt{->} \; T_n \; \texttt{->} \; T$
and $a_1 \; \texttt{::} \; T_1, \; \ldots, \; a_m \; \texttt{::} \; T_m$ and $m \leq n$
then $f \; a_1 \; \ldots \; a_m \; \texttt{::} \; T_{m+1} \; \texttt{->} \; \ldots \; \texttt{->} \; T_n \; \texttt{->} \; T$

6.7 More library functions

```
f . g  =  \x -> f (g x)
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(.) :: (b -> c) -> (a -> b) ->
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Example

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head2  =  head . tail
```

6.7 More library functions

```
(.) :: (b -> c) -> (a -> b) -> (a -> c)
f . g  =  \x -> f (g x)
```

Example

```
head2  =  head . tail

head2 [1,2,3]
```

```
const :: a -> (b -> a)
const x  =  \ _ -> x

curry :: ((a,b) -> c) -> (a -> b -> c)
curry f  =  \ x y -> f(x,y)
```

```
all :: (a -> Bool) -> [a] -> Bool
all p xs  =  and [p x | x <- xs]
```

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all :: (a -> Bool) -> [a] -> Bool
all p xs  =  and [p x | x <- xs]
```

Example

```
all (>1) [0, 1, 2]
= False
```

```
any :: (a -> Bool) -> [a] -> Bool
any p  =  or [p x | x <- xs]
```

```
akeWhile :: (a -> Bool) -> [a] -> [a]
```

```
takeWhile :: (a -> Bool) -> [a] -> [a]
takeWhile p []     = []
```