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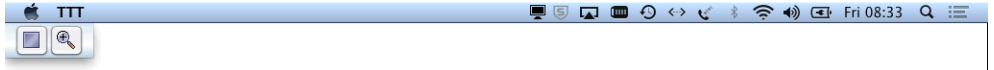

Informatik 2: Functional Programming

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Fakultät für Informatik
TU München

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Wintersemester 2014/15

October 9, 2014

Literatur

- Vorlesung orientiert sich stark an
  Thompson: *Haskell, the Craft of Functional Programming*

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  Thompson: *Haskell, the Craft of Functional Programming*
- Für Freunde der kompakten Darstellung:
  Hutton: *Programming in Haskell*

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- Notenbonus mit Hausaufgaben: siehe WWW-Seite
  Wer Hausaufgaben abschreibt oder abschreiben lässt, hat seinen Notenbonus sofort verwirkt.

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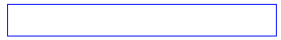

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- Zugang zu Piazza für Info 2 über Vorlesungsseite
- Funktioniert erst nach Anmeldung zur Übung

Haskell Installation

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- Bei Problemen mit der Installation des GHC:
  Zwei Beratungstermine, siehe Vorlesungsseite
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2. Functional Programming: The Idea

Functions are pure/mathematical functions:
Always same output for same input

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Computation = Application of functions to arguments

Example 1

In Haskell:

```
sum [1..10]
```

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In Java:

```
total = 0;
for (i = 1; i <= 10; ++i)
  total = total + i;
```

Example 2

In Haskell:

```
wellknown [] = []
wellknown (x:xs) = wellknown ys ++ [x] ++ wellknown zs
   where ys = [y | y <- xs, y <= x]
         zs = [z | z <- xs, x < z]
```

In Java:

```
void sort(int[] values) {
   if (values ==null || values.length==0){ return; }
   this.numbers = values;
   number = values.length;
   quicksort(0, number - 1);
}

void quicksort(int low, int high) {
   int i = low, j = high;
   int pivot = numbers[low + (high-low)/2];
   while (i <= j) {
      while (numbers[i] < pivot) { i++; }
      while (numbers[j] > pivot) { j--; }
      if (i <= j) {exchange(i, j); i++; j--; }
   }
   if (low < j) quicksort(low, j);
   if (i < high) quicksort(i, high);
}

void exchange(int i, int j) {
   int temp = numbers[i];
   numbers[i] = numbers[j];
   numbers[j] = temp;
}
```

There are two ways of constructing a software design:

One way is to make it so simple that there are obviously no deficiencies.

The other way is to make it so complicated that there are no obvious deficiencies.

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provable it's just (very basic) mathematics!

Aims of functional programming

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  not bits, bytes and pointers but whole data structures
- Minimize time to read and write programs:
  $\Rightarrow$ reduced development and maintenance time and costs
- Increased confidence in correctness of programs:
  clean and simple syntax and semantics
  $\Rightarrow$ programs are easier to
  - understand
  - test (Quickcheck!)
  - prove correct

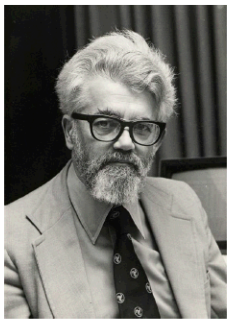
Historic Milestones

1930s

Alonzo Church develops the lambda calculus,
the core of all functional programming languages.

Historic Milestones

1950s



John McCarthy (Turing Award 1971) develops Lisp, the first functional programming language.

Historic Milestones

1970s

Robin Milner (FRS, Turing Award 1991) & Co. develop ML, the first modern functional programming language with *polymorphic types* and *type inference*.

Historic Milestones

1987

Haskell

A Purely Functional Language

An international committee of researchers initiates the development of Haskell, a standard lazy functional language.

Popular languages based on FP

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Scala (EPFL) = Java + FP

FP concepts in other languages

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Type classes:	C++ "concepts"

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- It gives you the edge over Millions of Java/C/C++ programmers out there
- FP concepts make you a better programmer, no matter which language you use
- To show you that programming need not be a black art with magic incantations like `public static void` but can be a science

3.1 Notational conventions

$e :: T$ means that expression e has type T

	Mathematics	Haskell
Function types:	$f : A \times B \to C$	`f :: A -> B -> C`

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Prefix binds stronger than infix:

`f a + b` means `(f a) + b`
not `f (a + b)`

3.2 Type `Bool`

Predefined: `True False not && || ==`

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This is an example of pattern matching.
The equations are tried in order. More later.

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GHCi answers

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+++ OK, passed 100 tests.
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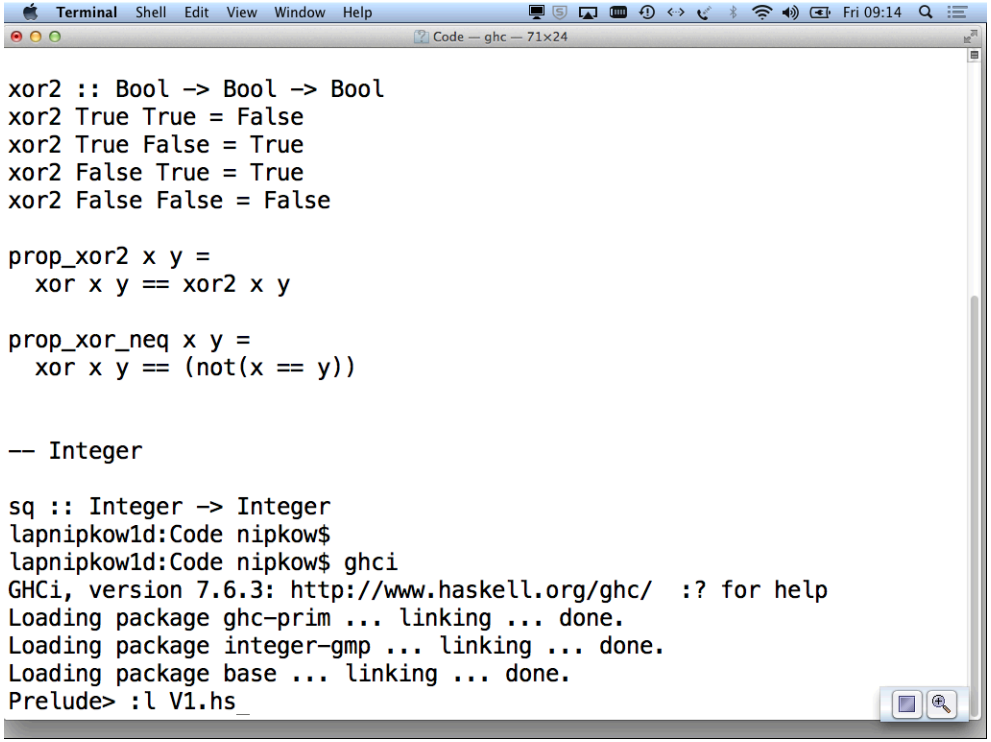
Typical test:

```
prop_f x y =
  f_efficient x y == f_naive x y
```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Code — ghc — 71×24
xor2 :: Bool -> Bool -> Bool
xor2 True True = False
xor2 True False = True
xor2 False True = True
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prop_xor2 x y =
xor x y == xor2 x y
prop_xor_neq x y =
xor x y == (not(x == y))
-- Integer
sq :: Integer -> Integer
lapnipkow1d:Code nipkow$
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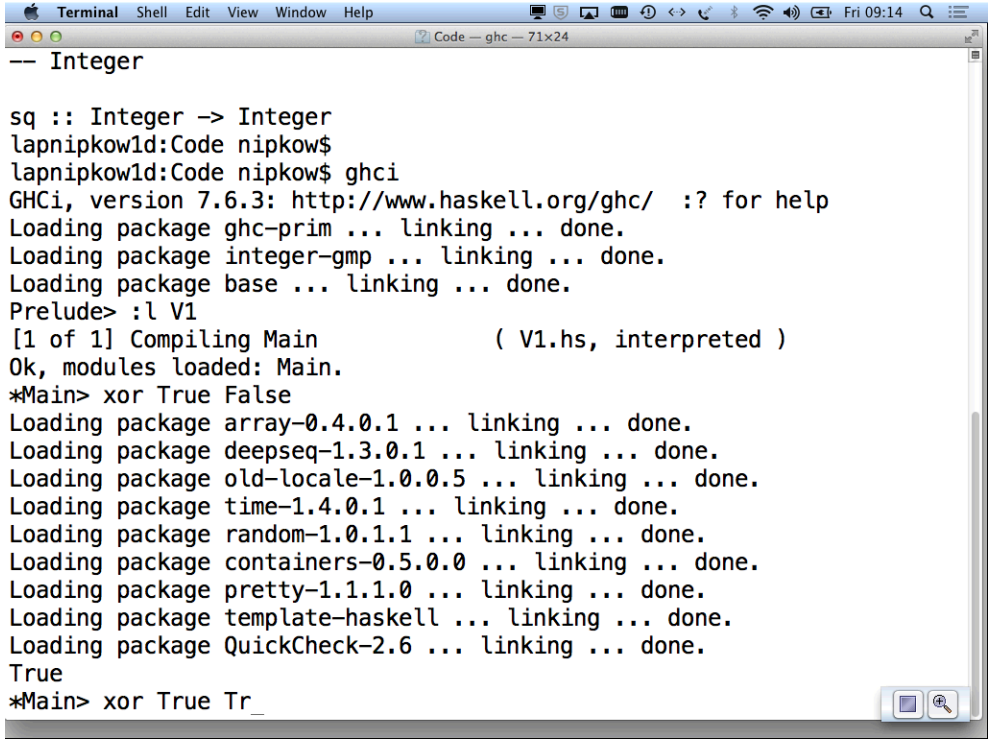
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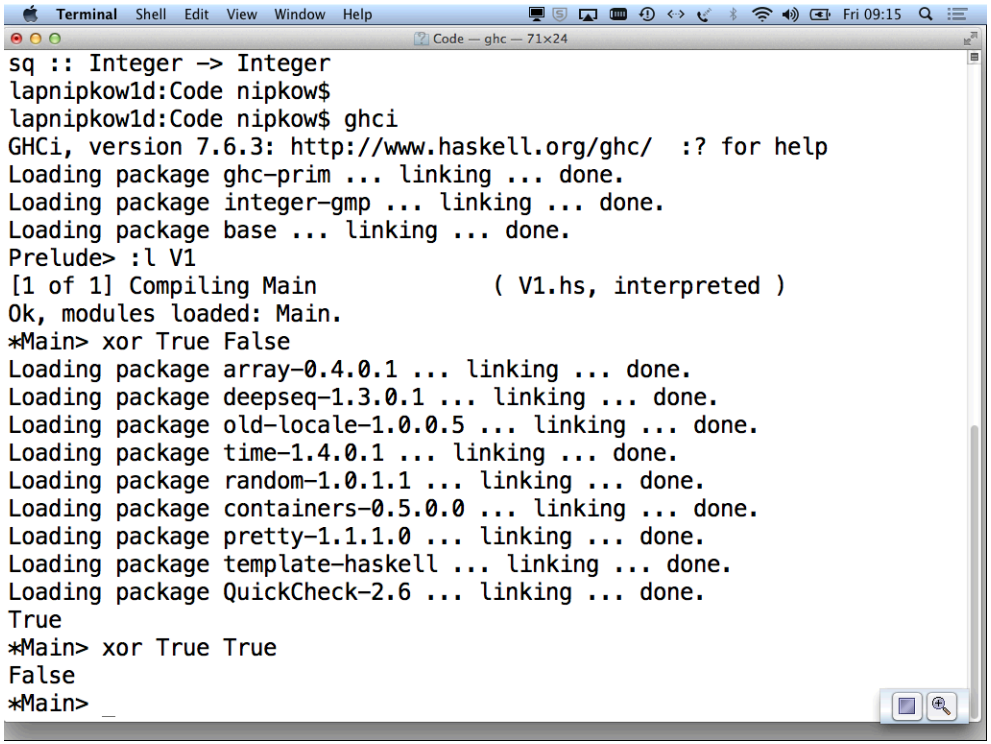
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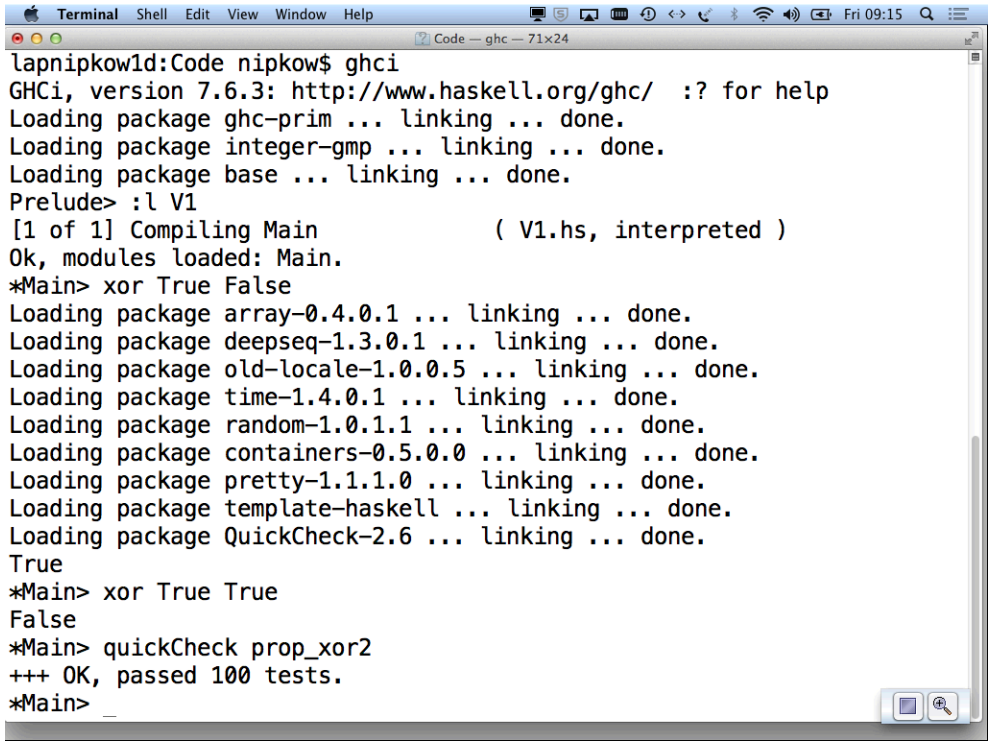
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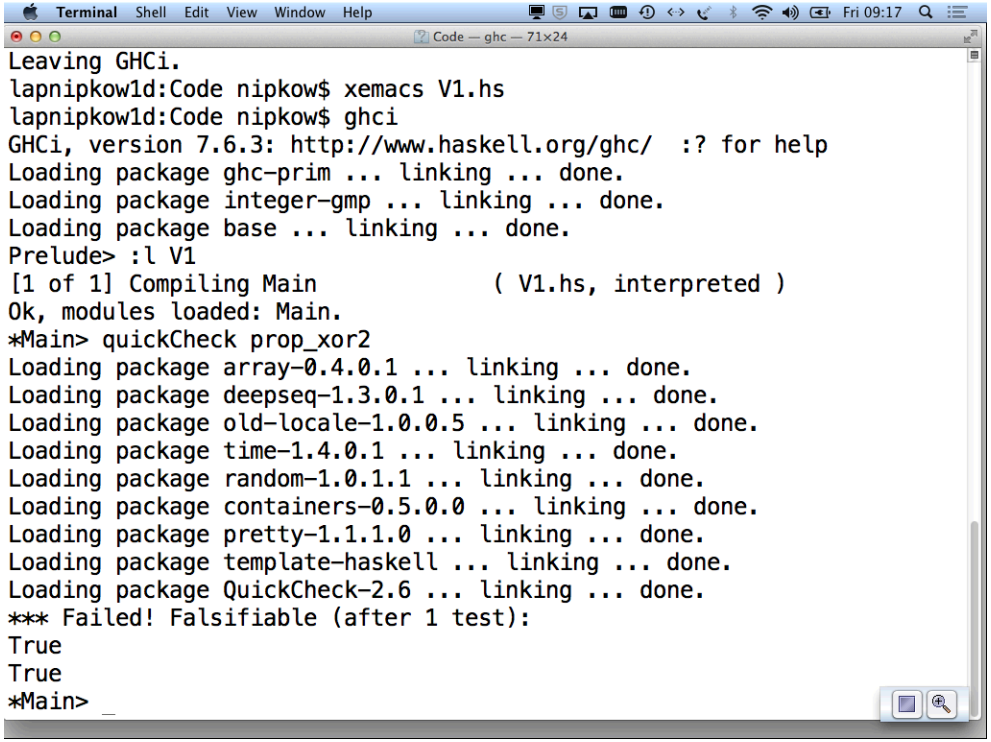

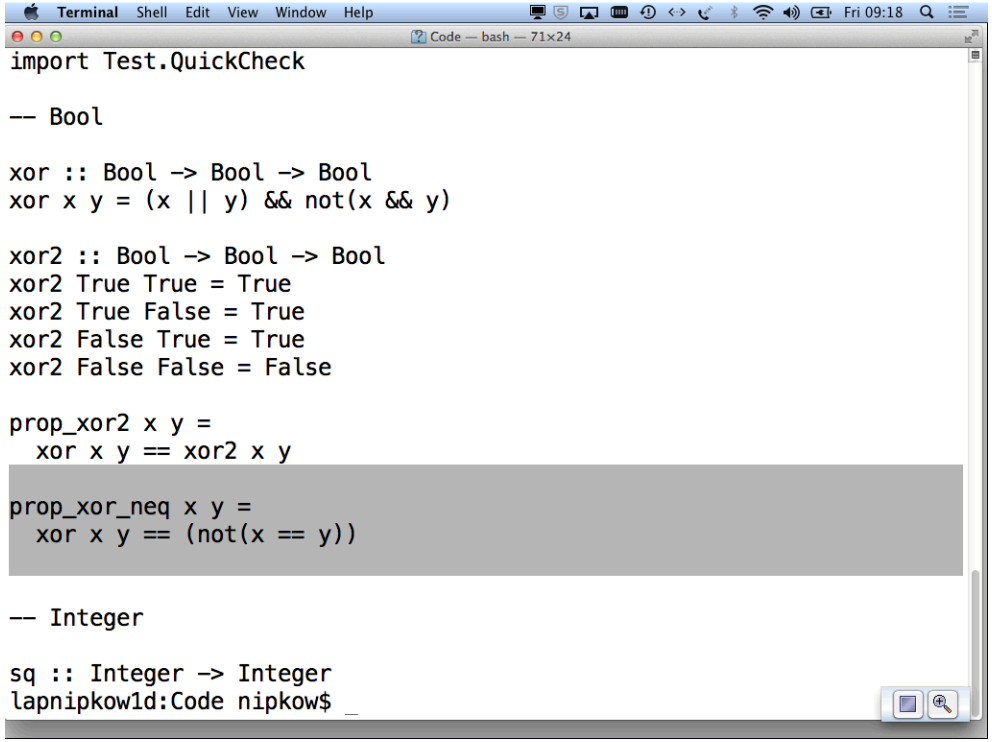

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Unlimited precision mathematical integers!
Predefined: `+ - * ^ div mod abs == /= < <= > >=`

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Warning:
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==, <= etc are overloaded and work on many types!

Example:

```
sq :: Integer -> Integer
sq n  =  n * n
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Evaluation:

```
sq (sq 3)
```

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```
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          = (3 * 3) * (3 * 3)
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sq (sq 3) = sq 3 * sq 3
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Evaluation of Haskell expressions
means
Using the defining equations from left to right.

3.4 Guarded equations

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Example: maximum of 2 integers.

```
max :: Integer -> Integer -> Integer
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Haskell also has `if-then-else`:

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Haskell also has `if-then-else`:

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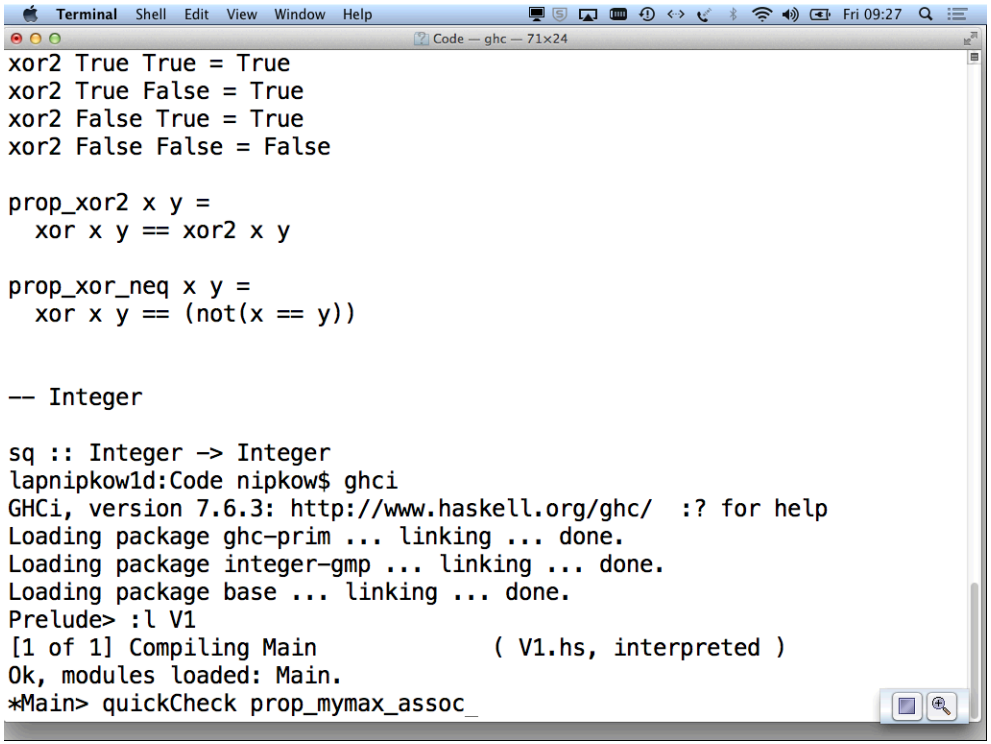
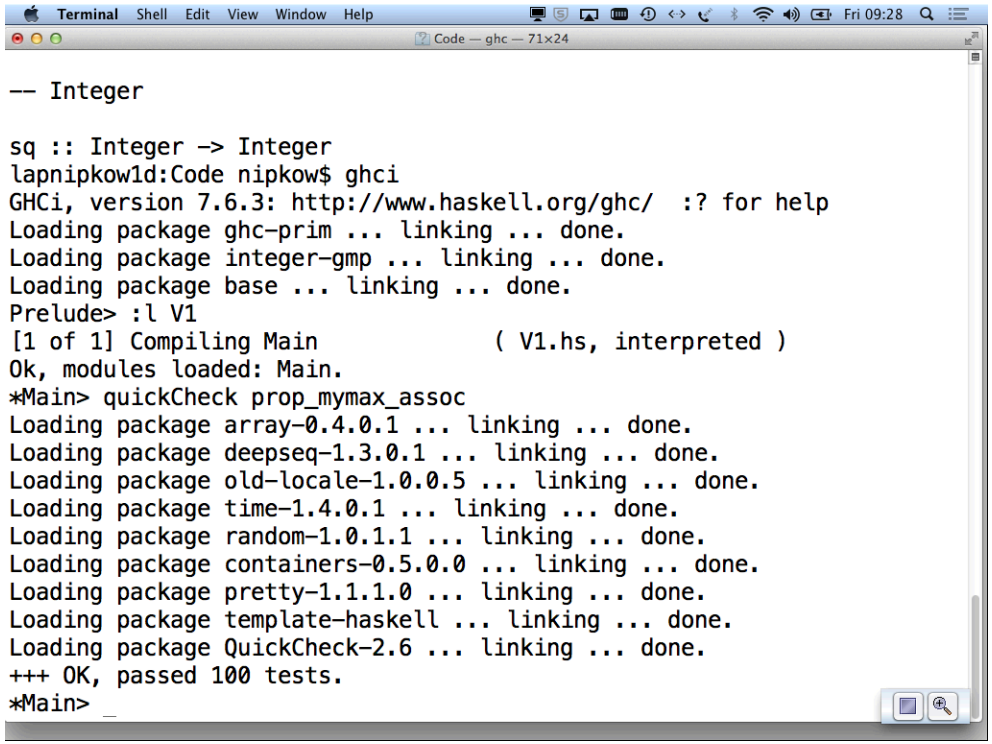
True?

```
prop_max_assoc x y z =
  max x (max y z) == max (max x y) z
```





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More compactly:

```
pow x 0  =  1
pow x n | n > 0  =  x * pow x (n-1)
```

Evaluating pow

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pow x 0  =  1
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```
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GHCi answers

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where `isDefined y  =  y == y`.

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The sum from 0 to n = n + (n-1) + (n-2) + ... + 0

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Typical recursion patterns for integers

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- other recursive calls, e.g. `f(n - 2)`
- also for negative numbers

Recursion in general

- Reduce a problem to a *smaller* problem,
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General problem solving strategy
in *any* programing language

3.6 Syntax matters

Functions are defined by one or more equations.
In the simplest case, each function is defined
by one (possibly conditional) equation:

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