Script **generated by TTT**

Title: Seidl: Programmoptimierung (28.11.2012)

Date: Wed Nov 28 09:35:33 CET 2012

Duration: 87:03 min

Pages: 57

solve x_2 eval x_2 x_3 solve x_3 eval x_3 x_1 solve x_1 eval x_1 x_3 solve x_3

stable!

$I[x_3] = \{x_1\}$

$\Rightarrow \quad \emptyset$

$\boxed{D[x_1] = \{a\}}$

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$\Rightarrow \quad \{a\}$

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$\boxed{\text{ok}}$

$I[x_3] = \{x_1, x_2\}$

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→ Evaluation starts with an interesting unknown x_i (e.g., the value at *stop*)

→ Then automatically all unknowns are evaluated which influence x_i :-)

→ The number of evaluations is often smaller than during worklist iteration ;-)

→ The algorithm is more complex but does not rely on pre-computation of variable dependencies :-))

→ It also works if variable dependencies during iteration change !!!

$\Longrightarrow$ interprocedural analysis

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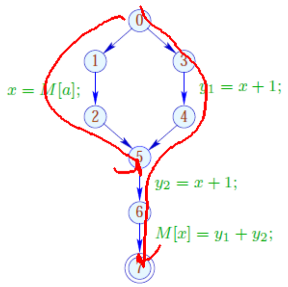
$\Longrightarrow$ interprocedural analysis

420

1.7 Eliminating Partial Redundancies

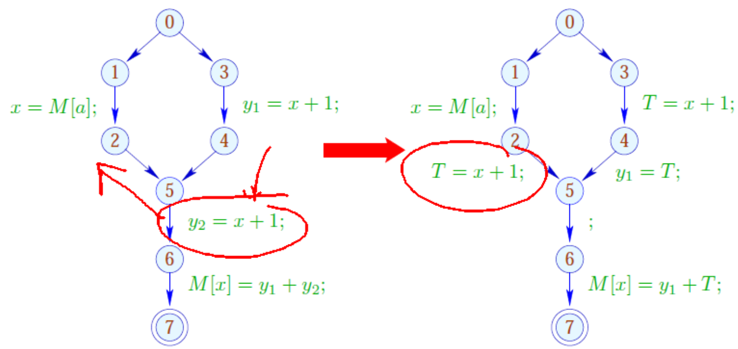
Example:



// $x + 1$ is evaluated on every path ...

// on one path, however, even twice :-(

Goal:



Idea:

(1) Insert assignments $T_e = e$; such that e is available at all points where the value of e is required.

(2) Thereby spare program points where e either is already available or will definitely be computed in future.

Expressions with the latter property are called very busy.

(3) Replace the original evaluations of e by accesses to the variable T_e.

$\Longrightarrow$ we require a novel analysis :-))

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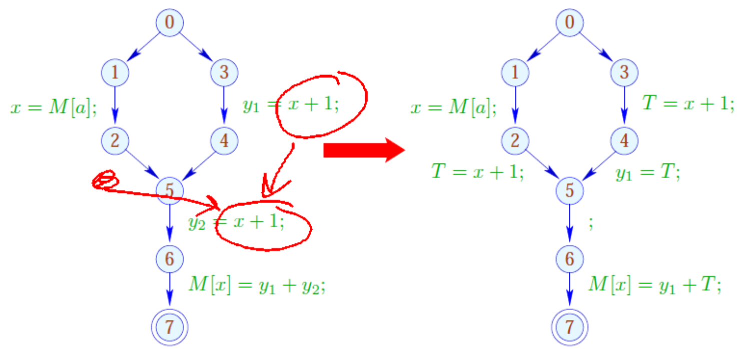

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An expression e is called busy along a path π , if the expression e is evaluated before any of the variables $x \in \mathit{Vars}(e)$ is overwritten.

// backward analysis!

e is called very busy at u , if e is busy along every path $\pi : u \to^* \mathit{stop}$.

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e is called very busy at u , if e is busy along every path $\pi : u \to^* \mathit{stop}$.

Accordingly, we require:

$$\mathcal{B}[u] \;=\; \bigcap \{ [\![\pi]\!]^\sharp\, \emptyset \mid \pi : u \to^* \mathit{stop} \}$$

where for $\pi = k_1 \ldots k_m$:

$$[\![\pi]\!]^\sharp \;=\; [\![k_1]\!]^\sharp \circ \ldots \circ [\![k_m]\!]^\sharp$$

Our complete lattice is given by:

$$\mathbb{B} = 2^{\mathit{Expr} \setminus \mathit{Vars}} \qquad \text{with} \quad \sqsubseteq \;=\; \supseteq$$

The effect $[\![k]\!]^\sharp$ of an edge $k = (u, \mathit{lab}, v)$ only depends on lab , i.e., $[\![k]\!]^\sharp = [\![\mathit{lab}]\!]^\sharp$ where:

$$\begin{array}{lcl}
[\![;]\!]^\sharp\, B & = & B \\
[\![\mathit{Pos}(e)]\!]^\sharp\, B & = & [\![\mathit{Neg}(e)]\!]^\sharp\, B \;=\; B \cup \{e\} \\
[\![x = e;]\!]^\sharp\, B & = & (B \setminus \mathit{Expr}_x) \cup \{e\} \\
[\![x = M[e];]\!]^\sharp\, B & = & (B \setminus \mathit{Expr}_x) \cup \{e\} \\
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\end{array}$$

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all $e, e_1, e_2 \notin Vars$

$x \cap a \cup b$

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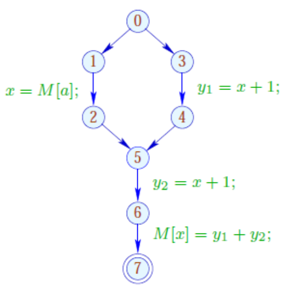
These effects are all distributive. Thus, the least solution of the constraint system yields precisely the MOP — given that *stop* is reachable from every program point :-)

Example:



7	$\emptyset$
6	$\{y_1 + y_2\}$
5	$\{x + 1\}$
4	$\{x + 1\}$
3	$\{x + 1\}$
2	$\{x + 1\}$
1	$\emptyset$
0	$\emptyset$

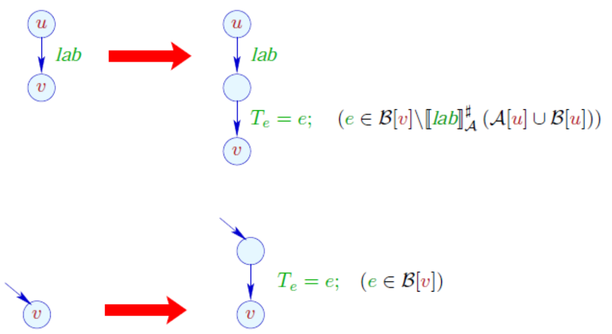
A point u is called safe for e, if $e \in \mathcal{A}[u] \cup \mathcal{B}[u]$, i.e., e is either available or very busy.

Idea:

- We insert computations of e such that e becomes available at all safe program points :-)
- We insert $T_e = e;$ after every edge (u, lab, v) with

$$e \in \mathcal{B}[v] \setminus [\![lab]\!]^\sharp_{\mathcal{A}}(\mathcal{A}[u] \cup \mathcal{B}[u])$$

Transformation 5.1:



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// analogously for the other uses of e

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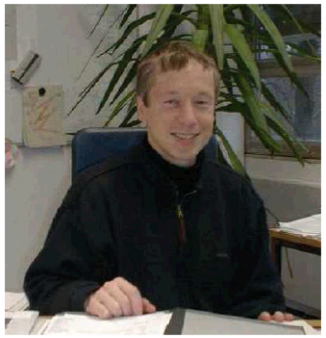
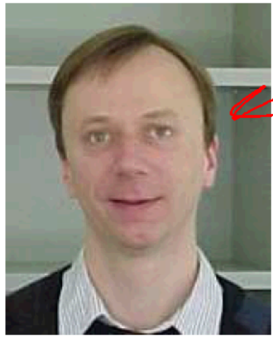


Bernhard Steffen, Dortmund



Jens Knoop, Wien

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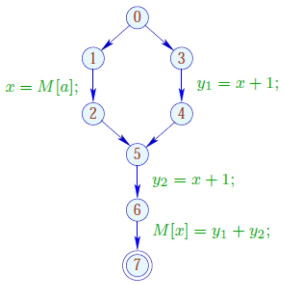

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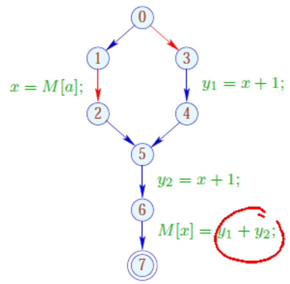

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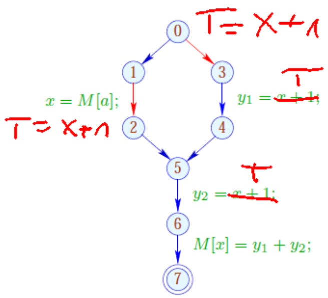

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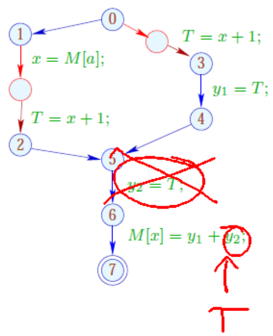

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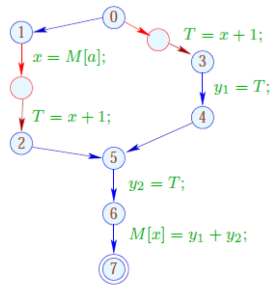

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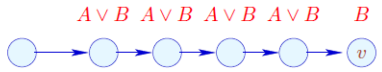
Correctness:

Let π denote a path reaching v after which a computation of an edge with e follows.

Then there is a maximal suffix of π such that for every edge $k = (u, lab, u')$ in the suffix:

$$e \in [\![lab]\!]^\sharp_{\mathcal{A}}(\mathcal{A}[u] \cup \mathcal{B}[u])$$



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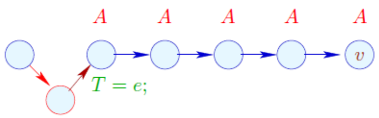
In particular, no variable in e receives a new value :-)

Then $T_e = e;$ is inserted before the suffix :-))



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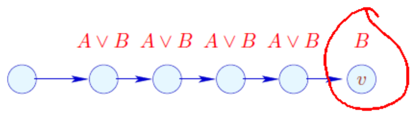

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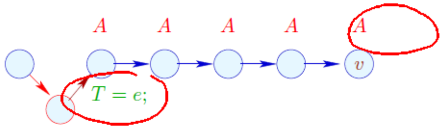


We conclude:

- Whenever the value of e is required, e is available :-)

  $\Longrightarrow$ correctness of the transformation

- Every $T = e;$ which is inserted into a path corresponds to an e which is replaced with T :-))

  $\Longrightarrow$ non-degradation of the efficiency

1.8 Application: Loop-invariant Code

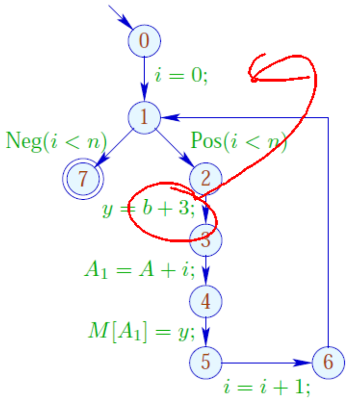
Example:

$$\text{for } (i = 0; i < n; i{+}{+})$$
$$a[i] = b + 3;$$

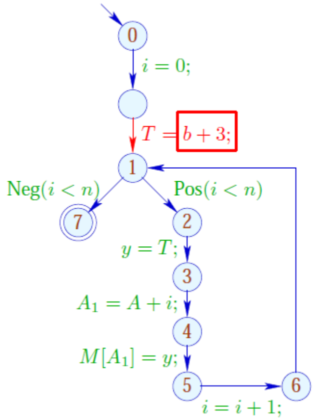
// The expression $b + 3$ is recomputed in every iteration :-(

// This should be avoided :-)

The Control-flow Graph:



Warning: $T = b + 3;$ may not be placed before the loop :



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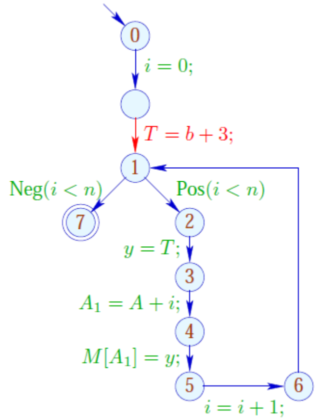

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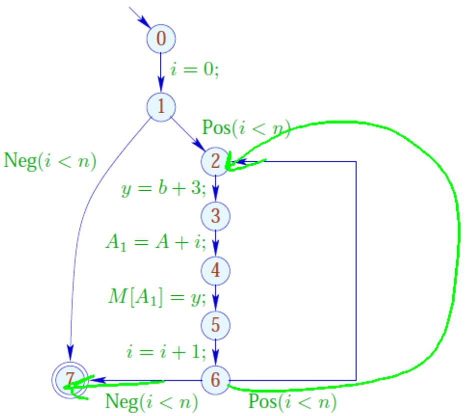
if(e) while(e) s

Idea: Transform into a do-while-loop ...



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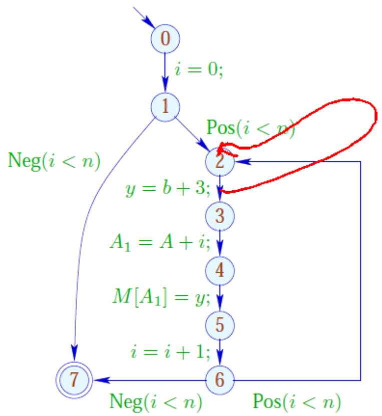
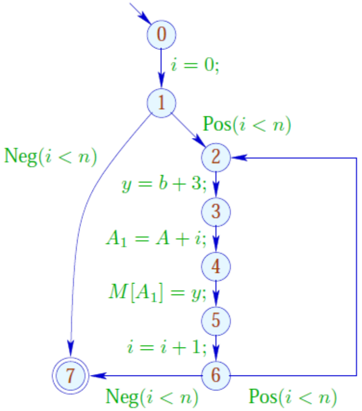


Application of T5 (PRE) :



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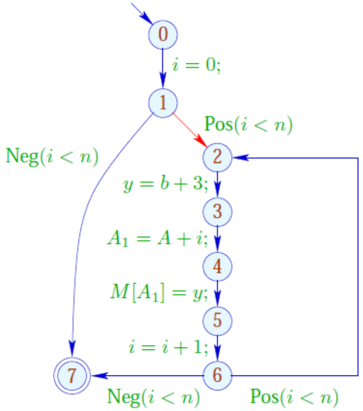

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Conclusion:

- Elimination of partial redundancies may move loop-invariant code out of the loop :-))
- This only works properly for do-while-loops :-(
- To optimize other loops, we transform them into do-while-loops before-hand:

$$\textsf{while}\ (b)\ stmt \implies \textsf{if}\ (b)\ \textsf{do}\ stmt\ \textsf{while}\ (b);$$

$\implies$ Loop Rotation

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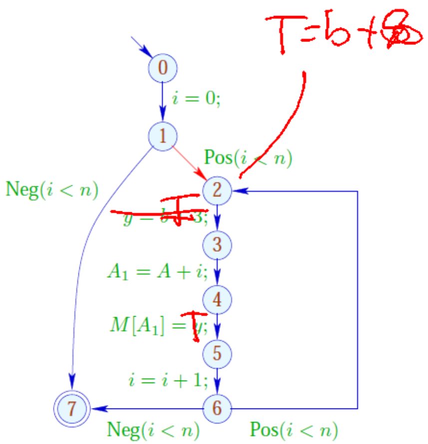

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$\Longrightarrow$ Loop Rotation

Problem:

If we do not have the source program at hand, we must re-construct potential loop headers ;-)

$\Longrightarrow$ Pre-dominators

u pre-dominates v, if every path $\pi : start \to^* v$ contains u. We write: $u \Rightarrow v$.

"$\Rightarrow$" is reflexive, transitive and anti-symmetric :-)

Computation:

We collect the nodes along paths by means of the analysis:

$$\mathbb{P} = 2^{Nodes} \quad , \qquad \sqsubseteq \; = \; \supseteq$$

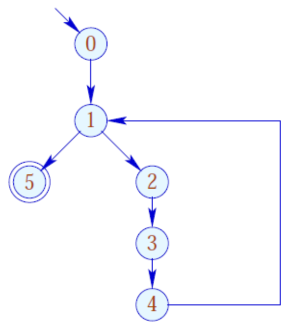
$$[\![(\_,\_,v)]\!]^\sharp\, P \quad = \quad P \cup \{v\}$$

Then the set $\mathcal{P}[v]$ of pre-dominators is given by:

$$\mathcal{P}[v] = \bigcap \{[\![\pi]\!]^\sharp\ \{start\} \mid \pi : start \to^* v\}$$

Since $[\![k]\!]^\sharp$ are distributive, the $\mathcal{P}[v]$ can computed by means of fixpoint iteration :-)

Example:



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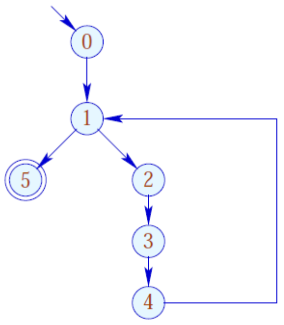

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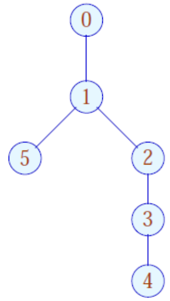
The partial ordering "$\Rightarrow$" in the example:



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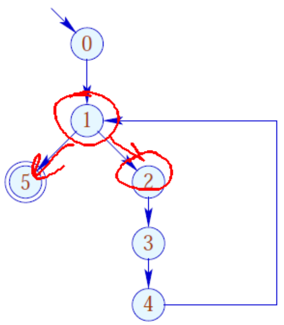

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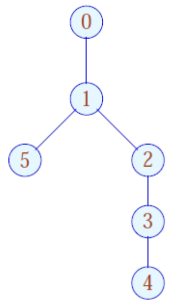

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Apparently, the result is a tree :-)

In fact, we have:

Theorem:

Every node v has at most one immediate pre-dominator.

Proof:

Assume:

there are $u_1 \neq u_2$ which immediately pre-dominate v.

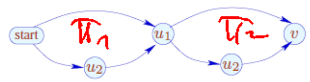
If $u_1 \Rightarrow u_2$ then u_1 not immediate.

Consequently, u_1, u_2 are incomparable :-)

Now for every $\pi : start \to^* v$:

$$\pi = \pi_1\ \pi_2 \qquad \text{with} \qquad \begin{array}{l} \pi_1 : start \to^* u_1 \\ \pi_2 : u_1 \to^* v \end{array}$$

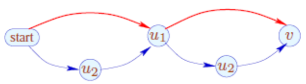
If, however, u_1, u_2 are incomparable, then there is path: $start \to^* v$ avoiding u_2 :



Now for every $\pi : start \to^* v$:

$$\pi = \pi_1\ \pi_2 \qquad \text{with} \qquad \begin{array}{l} \pi_1 : start \to^* u_1 \\ \pi_2 : u_1 \to^* v \end{array}$$

If, however, u_1, u_2 are incomparable, then there is path: $start \to^* v$ avoiding u_2 :



Now for every $\pi : start \to^* v$:

$$\pi = \pi_1\ \pi_2 \qquad \text{with} \qquad \begin{array}{l} \pi_1 : start \to^* u_1 \\ \pi_2 : u_1 \to^* v \end{array}$$

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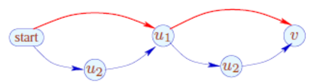


Observation:

The loop head of a while-loop pre-dominates every node in the body.

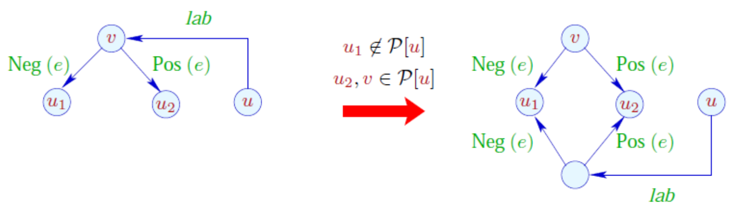
A back edge from the exit u to the loop head v can be identified through

$$v \in \mathcal{P}[u]$$

:-)

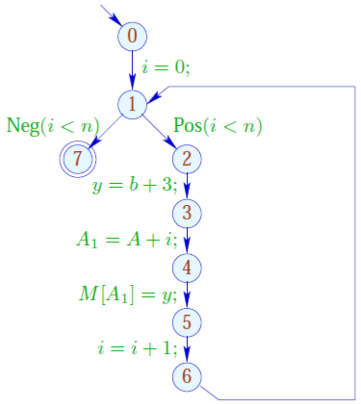
Accordingly, we define:

Transformation 6:

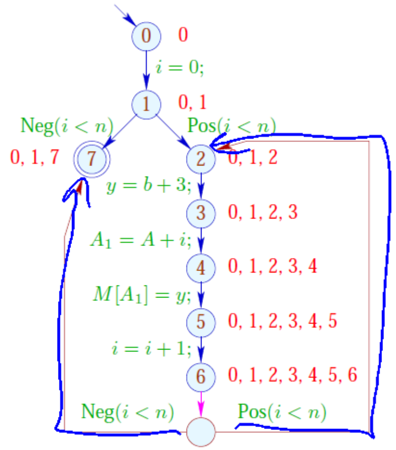


We duplicate the entry check to all back edges :-)

... in the Example:

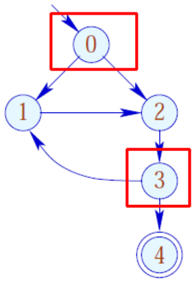


... in the Example:

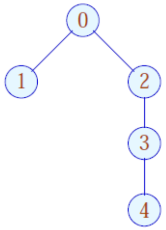


Warning:

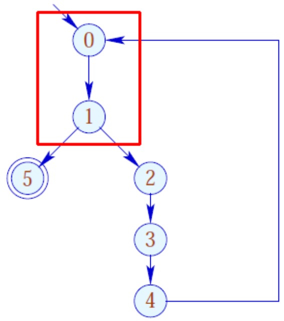
There are unusual loops which cannot be rotated:

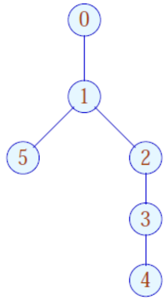


Pre-dominators:



... but also common ones which cannot be rotated:





Here, the complete block between back edge and conditional jump should be duplicated :-(