Script **generated by TTT**

Title: Seidl: Virtual_Machines (14.05.2013)

Date: Tue May 14 14:05:34 CEST 2013

Duration: 86:50 min

Pages: 43

- The optimization will cause Global Vectors to contain more components than just references to the free the variables that occur in one expression ...

Disadvantage: Superfluous components in Global Vectors prevent the deallocation of already useless heap objects ⟹ Space Leaks :-(

Potential Remedy: Deletion of references at the end of their life time.

---

22 Optimizations II: Closures

In some cases, the construction of closures can be avoided, namely for

- Basic values,
- Variables,
- Functions.

S, [ ]

Basic Values:

The construction of a closure for the value is at least as expensive as the construction of the B-object itself!

Therefore:

$$\text{code}_C\ b\ \rho\ \text{sd} \;=\; \text{code}_V\ b\ \rho\ \text{sd} \;=\; \begin{array}{l}\text{loadc b}\\ \text{mkbasic}\end{array}$$

This replaces:

```
mkvec 0           jump B     mkbasic     B:  ...
mkclos A      A:  loadc b    update
```

Variables:

Variables are either bound to values or to C-objects. Constructing another closure is therefore superfluous. Therefore:

$$\text{code}_C\ x\ \rho\ \text{sd} \quad = \quad \text{getvar}\ x\ \rho\ \text{sd}$$

This replaces:

```
getvar x ρ sd      mkclos A      A:   pushglob 0            update
mkvec 1            jump B             eval             B:   ...
```

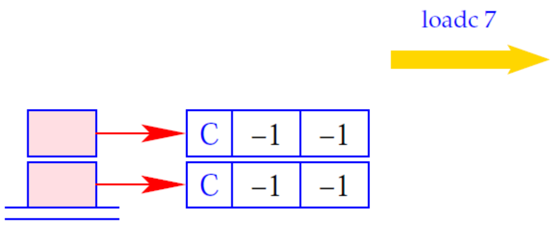
Example: $e \equiv \textbf{let rec}\ a = b\ \textbf{and}\ b = 7\ \textbf{in}\ a.$ $\text{code}_V\ e\ \emptyset\ 0$

produces:

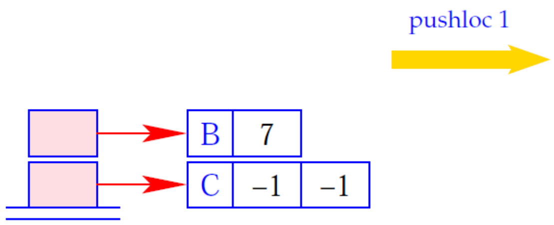
```
0   alloc 2      3   rewrite 2    3   mkbasic      2   pushloc 1
2   pushloc 0    2   loadc 7      3   rewrite 1    3   eval
                                                   3   slide 2
```

The execution of this instruction sequence should deliver the basic value 7 ...

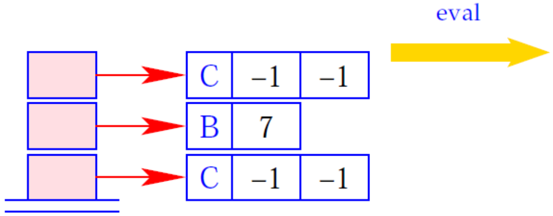
```
0   alloc 2      3   rewrite 2    3   mkbasic      2   pushloc 1
2   pushloc 0    2   loadc 7      3   rewrite 1    3   eval
                                                   3   slide 2
```



```
0   alloc 2      3   rewrite 2    3   mkbasic      2   pushloc 1
2   pushloc 0    2   loadc 7      3   rewrite 1    3   eval
                                                   3   slide 2
```



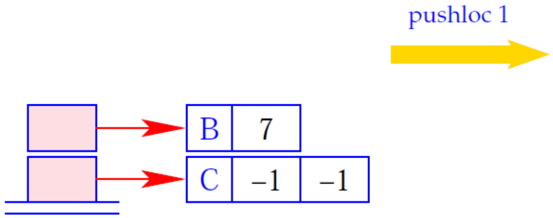
```
0   alloc 2      3   rewrite 2   3   mkbasic     2   pushloc 1
2   pushloc 0    2   loadc 7     3   rewrite 1   3   eval
                                                 3   slide 2
```



```
0   alloc 2      3   rewrite 2   3   mkbasic     2   pushloc 1
2   pushloc 0    2   loadc 7     3   rewrite 1   3   eval
                                                 3   slide 2
```

Segmentation Fault !!

```
0   alloc 2      3   rewrite 2   3   mkbasic     2   pushloc 1
2   pushloc 0    2   loadc 7     3   rewrite 1   3   eval
                                                 3   slide 2
```



Variables:

Variables are either bound to values or to C-objects. Constructing another closure is therefore superfluous. Therefore:

$$\text{code}_C\ x\ \rho\ \text{sd} \quad = \quad \text{getvar}\ x\ \rho\ \text{sd}$$

This replaces:

```
getvar x ρ sd      mkclos A     A:   pushglob 0         update
mkvec 1            jump B            eval          B:   ...
```

Example: $e \equiv \textbf{let rec}\ a = b\ \textbf{and}\ b = 7\ \textbf{in}\ a.$ $\text{code}_V\ e\ \emptyset\ 0$

produces:

```
0   alloc 2      3   rewrite 2   3   mkbasic     2   pushloc 1
2   pushloc 0    2   loadc 7     3   rewrite 1   3   eval
                                                 3   slide 2
```

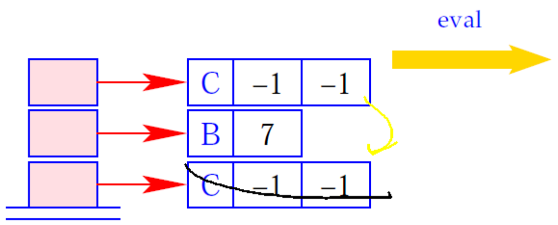
The execution of this instruction sequence should deliver the basic value 7 ...

```
0   alloc 2       3   rewrite 2     3   mkbasic       2   pushloc 1
2   pushloc 0     2   loadc 7       3   rewrite 1     3   eval
                                                      3   slide 2
```

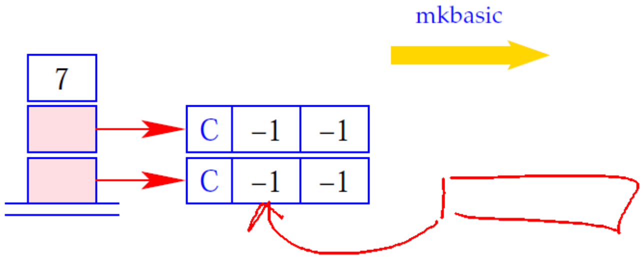
Segmentation Fault !!

```
0   alloc 2       3   rewrite 2     3   mkbasic       2   pushloc 1
2   pushloc 0     2   loadc 7       3   rewrite 1     3   eval
                                                      3   slide 2
```

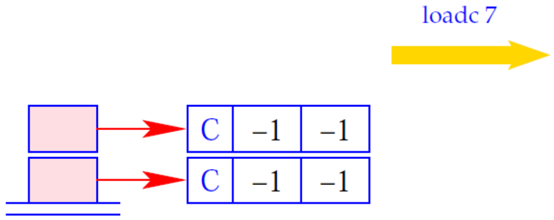
```
0   alloc 2       3   rewrite 2     3   mkbasic       2   pushloc 1
2   pushloc 0     2   loadc 7       3   rewrite 1     3   eval
                                                      3   slide 2
```



```
0   alloc 2       3   rewrite 2     3   mkbasic       2   pushloc 1
2   pushloc 0     2   loadc 7       3   rewrite 1     3   eval
                                                      3   slide 2
```



```
0   alloc 2       3   rewrite 2     3   mkbasic       2   pushloc 1
2   pushloc 0     2   loadc 7       3   rewrite 1     3   eval
                                                      3   slide 2
```



Functions:

Functions are values, which are not evaluated further. Instead of generating code that constructs a closure for an F-object, we generate code that constructs the F-object directly.

Therefore:

$$\text{code}_C\ (\textbf{fun}\ x_0 \ldots x_{k-1} \to e)\ \rho\ \text{sd} \quad = \quad \text{code}_V\ (\textbf{fun}\ x_0 \ldots x_{k-1} \to e)\ \rho\ \text{sd}$$

Functions:

Functions are values, which are not evaluated further. Instead of generating code that constructs a closure for an F-object, we generate code that constructs the F-object directly.

Therefore:

$$\text{code}_C\ (\textbf{fun}\ x_0 \ldots x_{k-1} \to e)\ \rho\ \text{sd} \quad = \quad \text{code}_V\ (\textbf{fun}\ x_0 \ldots x_{k-1} \to e)\ \rho\ \text{sd}$$

23 The Translation of a Program Expression

Execution of a program e starts with

$$\text{PC} = 0 \qquad \text{SP} = \text{FP} = \text{GP} = -1$$

The expression e must not contain free variables.

The value of e should be determined and then a halt instruction should be executed.

$$\begin{array}{lll} \text{code}\ e & = & \text{code}_V\ e\ \emptyset\ 0 \\ & & \text{halt} \end{array}$$

---

23 The Translation of a Program Expression

Execution of a program e starts with

$$\text{PC} = 0 \qquad \text{SP} = \text{FP} = \text{GP} = -1$$

The expression e must not contain free variables.

The value of e should be determined and then a halt instruction should be executed.

$$\begin{array}{lll} \text{code}\ e & = & \text{code}_V\ e\ \emptyset\ 0 \\ & & \text{halt} \end{array}$$

Remarks:

- The code schemata as defined so far produce Spaghetti code.
- Reason: Code for function bodies and closures placed directly behind the instructions mkfunval resp. mkclos with a jump over this code.
- Alternative: Place this code somewhere else, e.g. following the halt-instruction:

  Advantage: Elimination of the direct jumps following mkfunval and mkclos.

  Disadvantage: The code schemata are more complex as they would have to accumulate the code pieces in a Code-Dump.

$$\Longrightarrow$$

Solution:

Disentangle the Spaghetti code in a subsequent optimization phase :-)

Example:

let $a = 17$ **in let** $f =$ **fun** $b \to a + b$ **in** f 42

Disentanglement of the jumps produces:

```
0   loadc 17       2   mark B       3   B:  slide 2       1   pushloc 1
1   mkbasic        5   loadc 42     1       halt          2   eval
1   pushloc 0      6   mkbasic      0   A:  targ 1        2   getbasic
2   mkvec 1        6   pushloc 4    0       pushglob 0    2   add
2   mkfunval A     7   eval         1       eval          1   mkbasic
                   7   apply        1       getbasic      1   return 1
```

24 Structured Data

In the following, we extend our functional programming language by some datatypes.

24.1 Tuples

Constructors: $(.,\ldots,.)$, k-ary with $k \geq 0$;

Destructors: $\#j$ for $j \in \mathbb{N}_0$ (Projections)

We extend the syntax of expressions correspondingly:

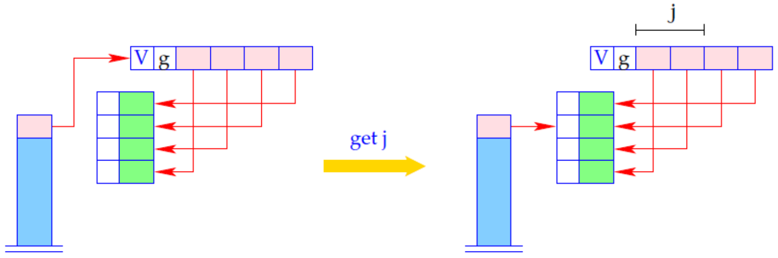
$$\begin{aligned} e \;::=\; & \ldots \mid (e_0,\ldots,e_{k-1}) \mid \#j\;e \\ & \mid \;\textbf{let}\;(x_0,\ldots,x_{k-1}) = e_1 \;\textbf{in}\; e_0 \end{aligned}$$

- In order to construct a tuple, we collect sequence of references on the stack. Then we construct a vector of these references in the heap using mkvec
- For returning components we use an indexed access into the tuple.

$$\begin{aligned} \mathrm{code}_V\;(e_0,\ldots,e_{k-1})\;\rho\;\mathrm{sd} \;=\;& \mathrm{code}_C\;e_0\;\rho\;\mathrm{sd} \\ & \mathrm{code}_C\;e_1\;\rho\;(\mathrm{sd}+1) \\ & \ldots \\ & \mathrm{code}_C\;e_{k-1}\;\rho\;(\mathrm{sd}+k-1) \\ & \mathrm{mkvec\;k} \end{aligned}$$

$$\begin{aligned} \mathrm{code}_V\;(\#j\;e)\;\rho\;\mathrm{sd} \;=\;& \mathrm{code}_V\;e\;\rho\;\mathrm{sd} \\ & \mathrm{get\;j} \\ & \mathrm{eval} \end{aligned}$$

In the case of CBV, we directly compute the values of the e_i.



```
if (S[SP] == (V,g,v))
   S[SP] = v[j];
else Error "Vector expected!";
```

Inversion: Accessing all components of a tuple simulataneously:

$$e \equiv \textbf{let}\;(\underset{\uparrow}{y_0},\ldots,\underset{\uparrow}{y_{k-1}}) = e_1\;\textbf{in}\;e_0$$

This is translated as follows:

$$\begin{aligned} \mathrm{code}_V\;e\;\rho\;\mathrm{sd} \;=\;& \mathrm{code}_V\;e_1\;\rho\;\mathrm{sd} \\ & \mathrm{getvec\;k} \\ & \mathrm{code}_V\;e_0\;\rho'\;(\mathrm{sd}+k) \\ & \mathrm{slide\;k} \end{aligned}$$

where $\rho' = \rho \oplus \{y_i \mapsto (L, sd+i+1) \mid i = 0,\ldots,k-1\}$.

The instruction getvec k pushes the components of a vector of length k onto the stack:

24.2 Lists

Lists are constructed by the constructors:

[] “Nil”, the empty list;

“::” “Cons”, right-associative, takes an element and a list.

Access to list components is possible by **match**-expressions ...

Example: The append function app:

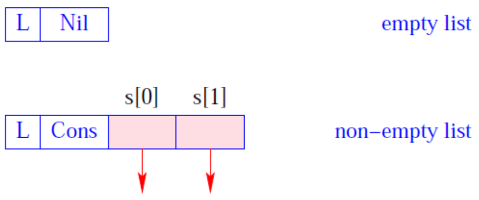
$$\begin{array}{lll} \text{app} & = & \textbf{fun}\ l\ y \rightarrow \textbf{match}\ l\ \textbf{with} \\ & & \qquad [\,] \quad \rightarrow \quad y \mid \\ & & \qquad h :: t \quad \rightarrow \quad h :: (\text{app}\ t\ y) \end{array}$$

---

accordingly, we extend the syntax of expressions:

$$\begin{array}{lll} e & ::= & \ldots \mid [\,] \mid (e_1 :: e_2) \\ & & \mid (\textbf{match}\ e_0\ \textbf{with}\ [\,] \rightarrow e_1 \mid h :: t \rightarrow e_2) \end{array}$$

Additionally, we need new heap objects:



24.3 Building Lists

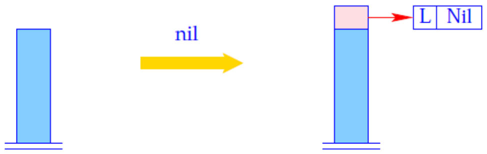
The new instructions nil and cons are introduced for building list nodes.

We translate for CBN:

$$\begin{array}{lll} \text{code}_V\ [\,]\ \rho\ \text{sd} & = & \text{nil} \\ \text{code}_V\ (e_1 :: e_2)\ \rho\ \text{sd} & = & \text{code}_C\ e_1\ \rho\ \text{sd} \\ & & \text{code}_C\ e_2\ \rho\ (\text{sd} + 1) \\ & & \text{cons} \end{array}$$

Note:

- With CBN: Closures are constructed for the arguments of “:”;
- With CBV: Arguments of “:” are evaluated :-)



SP++; S[SP] = new (L,Nil);

24.3 Building Lists

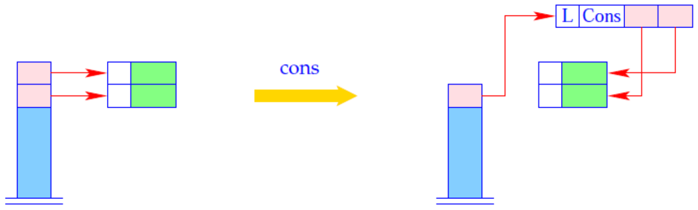
The new instructions nil and cons are introduced for building list nodes.

We translate for CBN:

$$\begin{aligned} \mathrm{code}_V\ []\ \rho\ \mathrm{sd} &= \mathrm{nil} \\ \mathrm{code}_V\ (e_1 :: e_2)\ \rho\ \mathrm{sd} &= \mathrm{code}_C\ e_1\ \rho\ \mathrm{sd} \\ &\quad\ \mathrm{code}_C\ e_2\ \rho\ (\mathrm{sd}+1) \\ &\quad\ \mathrm{cons} \end{aligned}$$

Note:

- With CBN: Closures are constructed for the arguments of ":";
- With CBV: Arguments of ":" are evaluated :-)



S[SP-1] = new (L,Cons, S[SP-1], S[SP]);
SP- -;

24.3 Building Lists

The new instructions nil and cons are introduced for building list nodes.

We translate for CBN:

$$\begin{aligned} \mathrm{code}_V\ []\ \rho\ \mathrm{sd} &= \mathrm{nil} \\ \mathrm{code}_V\ (e_1 :: e_2)\ \rho\ \mathrm{sd} &= \mathrm{code}_V\ e_1\ \rho\ \mathrm{sd} \quad ✓ \\ &\quad\ \mathrm{code}_V\ e_2\ \rho\ (\mathrm{sd}+1) \\ &\quad\ \mathrm{cons} \end{aligned}$$

Note:

- With CBN: Closures are constructed for the arguments of ":";
- With CBV: Arguments of ":" are evaluated :-)

24.4 Pattern Matching

Consider the expression $e \equiv \textbf{match}\ e_0\ \textbf{with}\ [] \to e_1 \mid h :: t \to e_2.$

Evaluation of e requires:

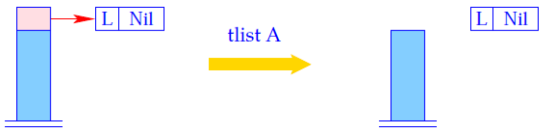
- evaluation of e_0;
- check, whether resulting value v is an L-object;
- if v is the empty list, evaluation of e_1 ...
- otherwise storing the two references of v on the stack and evaluation of e_2. This corresponds to binding h and t to the two components of v.

In consequence, we obtain (for CBN as for CBV):

$$\begin{array}{rcrl} \text{code}_V\; e\; \rho\; \text{sd} & = & & \text{code}_V\; e_0\; \rho\; \text{sd} \\ & & & \text{tlist A} \\ & & & \text{code}_V\; e_1\; \rho\; \text{sd} \\ & & & \text{jump B} \\ & & \text{A :} & \text{code}_V\; e_2\; \rho'\; (\text{sd}+2) \\ & & & \text{slide } 2 \\ & & \text{B :} & \ldots \end{array}$$

where $\rho' = \rho \oplus \{h \mapsto (L, sd+1), t \mapsto (L, sd+2)\}$.

The new instruction tlist A does the necessary checks and (in the case of Cons) allocates two new local variables:



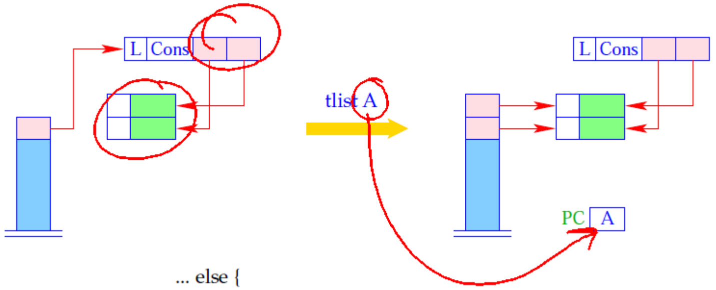
```
h = S[SP];
if (H[h] != (L,...)
    Error “no list!”;
if (H[h] == (_,Nil)) SP- -;
    ...
```

In consequence, we obtain (for CBN as for CBV):

$$\begin{array}{rcrl} \text{code}_V\; e\; \rho\; \text{sd} & = & & \text{code}_V\; e_0\; \rho\; \text{sd} \\ & & & \text{tlist A} \\ & & & \text{code}_V\; e_1\; \rho\; \text{sd} \\ & & & \text{jump B} \\ & & \text{A :} & \text{code}_V\; e_2\; \rho'\; (\text{sd}+2) \\ & & & \text{slide } 2 \\ & & \text{B :} & \ldots \end{array}$$

where $\rho' = \rho \oplus \{h \mapsto (L, sd+1), t \mapsto (L, sd+2)\}$.

The new instruction tlist A does the necessary checks and (in the case of Cons) allocates two new local variables:



```
... else {
  S[SP+1] = S[SP]→s[1];
  S[SP] = S[SP]→s[0];
  SP++; PC = A;
}
```

Example: The (disentangled) body of the function app with app $\mapsto (G,0)$:

```
0        targ 2        3       pushglob 0     0   C:  mark D
0        pushloc 0     4       pushloc 2      3       pushglob 2
1        eval          5       pushloc 6      4       pushglob 1
1        tlist A       6       mkvec 3        5       pushglob 0
0        pushloc 1     4       mkclos C       6       eval
1        eval          4       cons           6       apply
1        jump B        3       slide 2        1   D:  update
2   A:   pushloc 1     1   B:  return 2
```

Note:

Datatypes with more than two constructors need a generalization of the tlist instruction, corresponding to a `switch`-instruction :-)

24.5 Closures of Tuples and Lists

The general schema for code_C can be optimized for tuples and lists:

$$\begin{aligned}
\text{code}_C\ (e_0,\ldots,e_{k-1})\ \rho\ \text{sd} &= \text{code}_V\ (e_0,\ldots,e_{k-1})\ \rho\ \text{sd} &&= \text{code}_C\ e_0\ \rho\ \text{sd}\\
&&&\ \text{code}_C\ e_1\ \rho\ (\text{sd}+1)\\
&&&\ \ldots\\
&&&\ \text{code}_C\ e_{k-1}\ \rho\ (\text{sd}+k-1)\\
&&&\ \text{mkvec k}\\
\text{code}_C\ []\ \rho\ \text{sd} &= \text{code}_V\ []\ \rho\ \text{sd} &&= \text{nil}\\
\text{code}_C\ (e_1 : e_2)\ \rho\ \text{sd} &= \text{code}_V\ (e_1 : e_2)\ \rho\ \text{sd} &&= \text{code}_C\ e_1\ \rho\ \text{sd}\\
&&&\ \text{code}_C\ e_2\ \rho\ (\text{sd}+1)\\
&&&\ \text{cons}
\end{aligned}$$

24.5 Closures of Tuples and Lists

The general schema for code_C can be optimized for tuples and lists:

$$\begin{aligned}
\text{code}_C\ (e_0,\ldots,e_{k-1})\ \rho\ \text{sd} &= \text{code}_V\ (e_0,\ldots,e_{k-1})\ \rho\ \text{sd} &&= \text{code}_C\ e_0\ \rho\ \text{sd}\\
&&&\ \text{code}_C\ e_1\ \rho\ (\text{sd}+1)\\
&&&\ \ldots\\
&&&\ \text{code}_C\ e_{k-1}\ \rho\ (\text{sd}+k-1)\\
&&&\ \text{mkvec k}\\
\text{code}_C\ []\ \rho\ \text{sd} &= \text{code}_V\ []\ \rho\ \text{sd} &&= \text{nil}\\
\text{code}_C\ (e_1 : e_2)\ \rho\ \text{sd} &= \text{code}_V\ (e_1 : e_2)\ \rho\ \text{sd} &&= \text{code}_C\ e_1\ \rho\ \text{sd}\\
&&&\ \text{code}_C\ e_2\ \rho\ (\text{sd}+1)\\
&&&\ \text{cons}
\end{aligned}$$

---

25 Last Calls

A function application is called last call in an expression e if this application could deliver the value for e.

A last call usually is the outermost application of a defining expression.

A function definition is called tail recursive if all recursive calls are last calls.

Examples:

$r\ t\ (h :: y)$ is a last call in $\quad$ $\textbf{match}\ x\ \textbf{with}\ []\ \to y \mid h :: t\ \to r\ t\ (h :: y)$

$f\ (x-1)$ is not a last call in $\quad$ $\textbf{if}\ x \leq 1\ \textbf{then}\ 1\ \textbf{else}\ x * f\ (x-1)$

Observation: Last calls in a function body need no new stack frame!

$\Longrightarrow$

Automatic transformation of tail recursion into loops!!!