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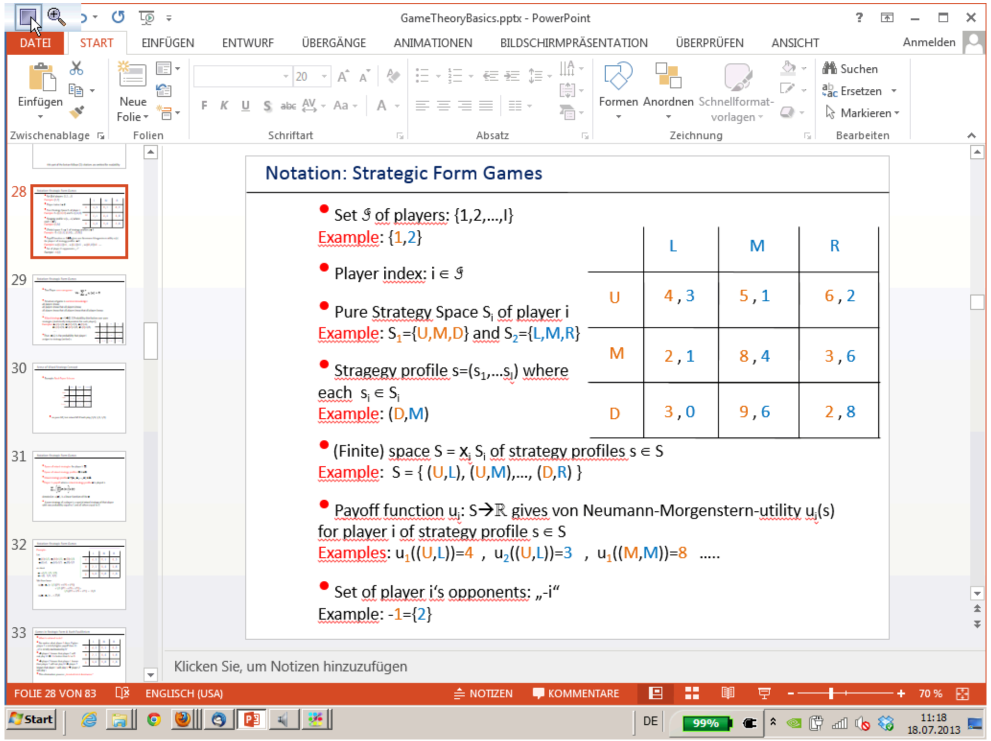

Notation: Strategic Form Games

- Set $\mathfrak{I}$ of players: {1,2,...,I}
Example: {1,2}
- Player index: i ∈ $\mathfrak{I}$
- Pure Strategy Space S_i of player i
Example: S_1={U,M,D} and S_2={L,M,R}
- Stragegy profile s=(s_1,...s_I) where each $s_i \in S_i$
Example: (D,M)
- (Finite) space S = x_i S_i of strategy profiles s ∈ S
Example: S = { (U,L), (U,M),..., (D,R) }
- Payoff function u_i: S→ℝ gives von Neumann-Morgenstern-utility $u_i(s)$ for player i of strategy profile s ∈ S
Examples: $u_1((U,L))=4$, $u_2((U,L))=3$, $u_1((M,M))=8$ .....
- Set of player i's opponents: „-i“
Example: -1={2}

	L	M	R
U	4 , 3	5 , 1	6 , 2
M	2 , 1	8 , 4	3 , 6
D	3 , 0	9 , 6	2 , 8

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Games in Strategic Form & Nash Equilibrium

More Notation:

- Discussing player i's strategy-options, holding other player's options fixed:
- $s_{-i} \in S_{-i}$: „other player's strategies“
- Short notation: $(s'_i, s_{-i}) := (s_1, \ldots, s_{i-1}, s'_i, s_{i+1}, \ldots, s_I)$
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Definition:

- Pure strategy s_i is strictly dominated for player if σ'_i exists so that $u_i(\sigma'_i, s_{-i}) > u_i(s_i, s_{-i})$ for all $s_{-i} \in S_{-i}$
- … weakly dominated: $u_i(\sigma'_i, s_{-i}) \geq u_i(s_i, s_{-i})$ for all $s_{-i} \in S_{-i}$ (and > for at least one s_{-i})
- If $u_i(\sigma'_i, s_{-i}) > u_i(s_i, s_{-i})$ for all $s_{-i} \in S_{-i}$ we also have $u_i(\sigma'_i, \sigma_{-i}) > u_i(s_i, \sigma_{-i})$ for all $\sigma_{-i} \in S_{-i}$ because $u_i(\sigma'_i, \sigma_{-i})$ is a convex function of $u_i(\sigma'_i, s_{-i}), u_i(\sigma'_i, s'_{-i}), u_i(\sigma'_i, s''_{-i}), \ldots$.

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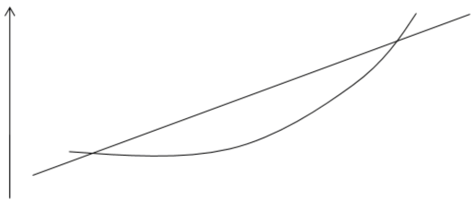
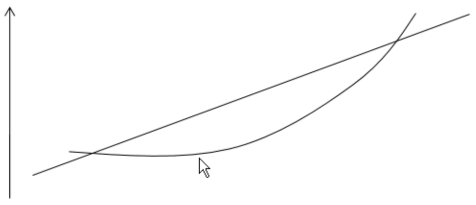
Strictly Convex function:

$f(tx+(1-t)y) < tf(x) +(1-t)f(y)$





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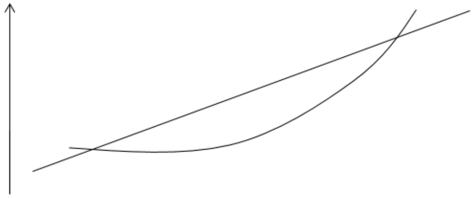

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Games in Strategic Form & Nash Equilibrium

- What about dominated mixed strategies?
- Easy: A mixed strategy that assigns positive probabilities to pure strategies that are dominated is dominated
- But: A mixed strategy may be dominated even if it assigns positive probabilities to pure strategies that are not even weakly dominated:

	L	R
U	1, 3	-2, 0
M	-2, 0	1, 3
D	0, 1	0, 1

Example:

- U and M are not dominated by D for player 1
- But: Playing $\sigma_1=(½, ½, 0)$ gives expected utility $u_1(\sigma_1, *) = -1/2$ no matter what 2 plays → D $(\sigma_D=(0, 0, 1))$ dominates σ_1

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A note on rationality

	L	R
U	8, 10	-100, 9
D	7, 6	6, 5

- Iterated strict dominance → (U,L)
- BUT: psychology → play D instead of U because „U is unsafe“

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Game Theory ←→ Decision Theory

- Example
- Iterated strict dominance → (U,L)

	L	R
U	1, 3	4, 1
D	0, 2	3, 4

- If player 1 reduces his payoff for U by 2:
  - decison theory: no use
  - game theory: new iterated strict dominance → (D,R)

	L	R
U	-1, 3	2, 1
D	0, 2	3, 4

Games in Strategic Form & Nash Equilibrium

Vickrey Auction & Iterated dominance

- Good‘s valuations: v_i ; Assume common knowledge for the moment
- Bids: s_i
- Second price:
  - winning condition: $s_i > \max_{j\neq i} s_j$
  - let $r_i := \max_{j\neq i} s_j$ r_i is the price having to be paid
  - winner i ´s utility: $u_i = v_i - r_i$; other players utility = 0
- for each player bidding true valuation is weakly dominant:
  - case $s_i > v_i$: (overbidding)
    - If $r_i > s_i$: looses → $u_i = 0$
      → could have bidden v_i as well
    - If $r_i \leq v_i$: wins → $u_i = v_i - r_i$
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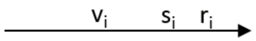

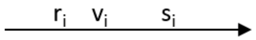

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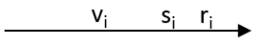

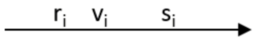

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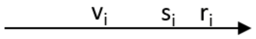

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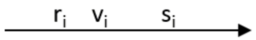
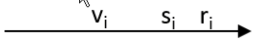


Games in Strategic Form & Nash Equilibrium

Vickrey Auction & Iterated dominance

- Good's valuations: v_i ; Assume common knowledge for the moment
- Bids: s_i
- Second price:
  - winning condition: $s_i > \max_{j \neq i} s_j$
  - let $r_i := \max_{j \neq i} s_j$ r_i is the price having to be paid
  - winner i ´s utility: $u_i = v_i - r_i$; other players utility = 0

- for each player bidding true valuation is weakly dominant:
  - case $s_i > v_i$: (overbidding)
    - If $r_i > s_i$: looses → $u_i = 0$
      → could have bidden v_i as well



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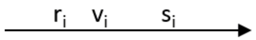

Games in Strategic Form & Nash Equilibrium

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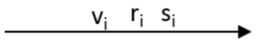
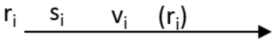


- case $v_i < r_i < s_i$:
  - i wins → $u_i = v_i - r_i < 0$ (winner's curse)
  → should have bidden $v_i = r_i$ → $u_i = 0$ at least
- case $s_i < v_i$: (underbidding)



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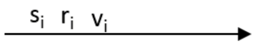

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Games in Strategic Form & Nash Equilibrium

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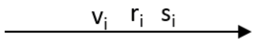

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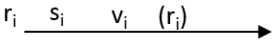

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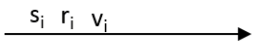

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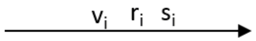

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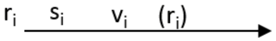

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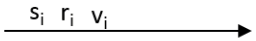

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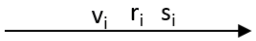

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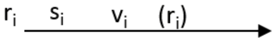

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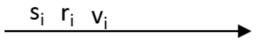

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Games in Strategic Form & Nash Equilibrium

Nash Equilibrium

- Nash Equilibrium : strategy profile: each player‘s strategy is optimal response to all other player‘s strategies:

- Mixed strategy profile σ* is Nash Equilibrium if

for all i: $u_i(\sigma^*_i, \sigma^*_{-i}) \geq u_i(s_i, \sigma^*_{-i})$ for all $s_i \in S_i$

(Pure strategy profiles also possible → „pure strategy NE“)

- Strategy profile s* is Strict Nash Equilibrium: if it is a NE and

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Strict NE is necessarily a pure strategy NE by definition.

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→ in NE def we must only check for pure alternatives s_i

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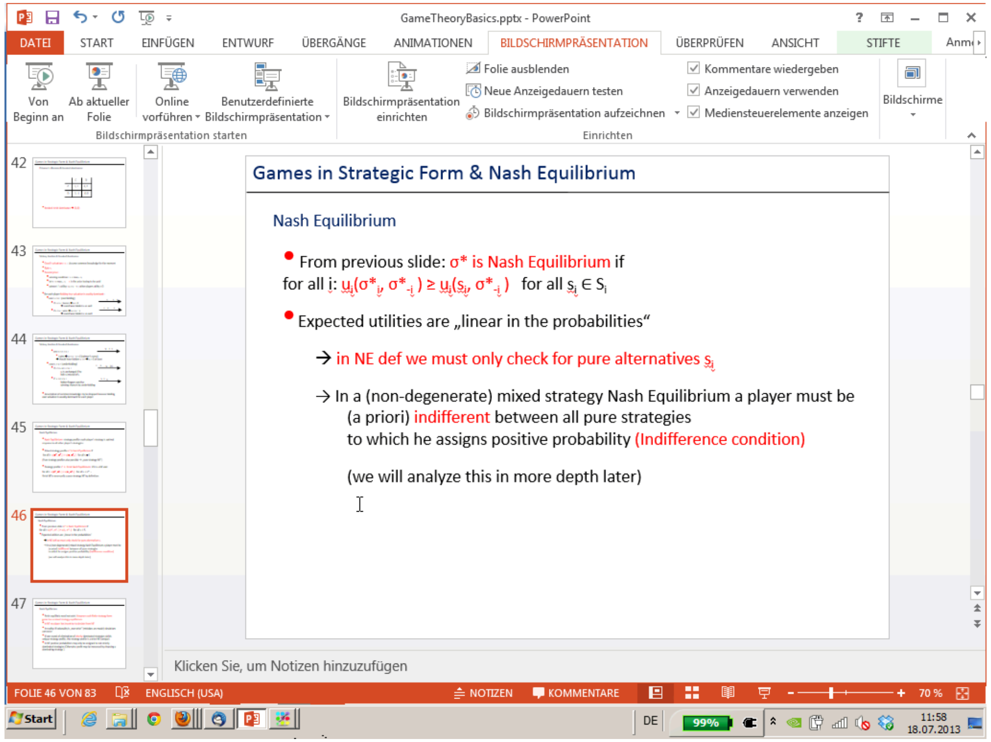

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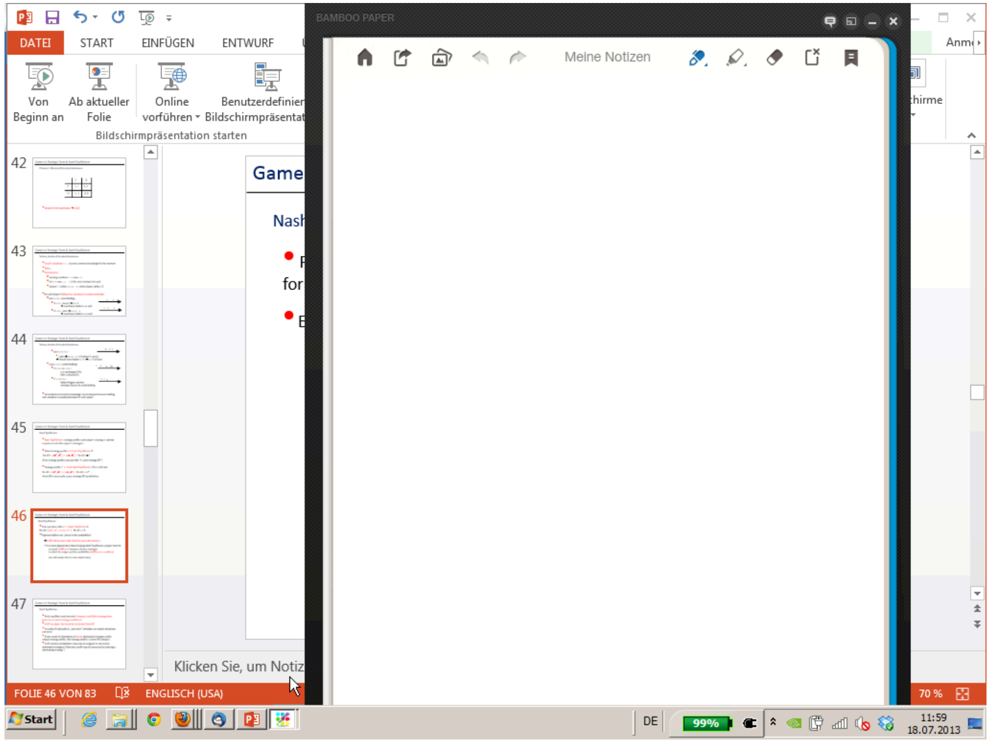

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Games in Strategic Form & Nash Equilibrium

Nash Equilibrium

- Strict equilibria need not exist. However each finite strategy form game has a mixed strategy equilibrium.
- In NE no player has incentive to deviate from NE
- In reality: If rationality is „non-strict“ (mistakes are made): deviations can occur
- If one round of elimination of strictly dominated strategies yields unique strategy profile, this strategy profile is a strict NE (unique)
- In NE positive probabilities may only be assigned to not-strictly dominated strategies (Otherwise profit may be increased by choosing a dominating strategy).

Games in Strategic Form & Nash Equilibrium

Nash Equilibrium: Example: Cournot Competition

- Cournot model: Duopoly. Each of two firms (players) i produces same good.
- Output levels q_i are chosen from sets Q_i
- Cost of production is $c_i(q_i)$
- Market price is $p(q) = p(q_1+q_2)$
- Firm i‘s profit is then $u_i(q_1, q_2) = q_i\, p(q) - c_i(q_i)$
- Cournot reaction functions $r_1 : Q_2 \rightarrow Q_1$ and $r_2 : Q_1 \rightarrow Q_2$ specify optimal reaction on output level of opponent

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Games in Strategic Form & Nash Equilibrium

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- Under certain reasonable assumptions (see [1]) we can maximize e.g. $u_2(q_1, q_2)$ by solving $d/dq_2\ u_2 (q_1, q_2) = 0$ which yields

$d/dq_2\ [q_2\, p(q_1 ,q_2) - c_2(q_2)] = p(q_1 ,q_2) + p'(q_1 ,q_2)\, q_2 - c_2'(q_2) = 0.$

Inserting $r_2 (q_1)$ for q_2

$p(q_1 + r_2 (q_1)) + p'(q_1 + r_2 (q_1))\, r_2 (q_1) – c_2'(r_2 (q_1)) = 0$

gives the defining equation for $r_2 (.)$.

(analogous for $r_1 (.)$).

- The intersections of the functions r_2 and r_1 are the NE of the Cournot game.
- Example: Linear demand $p(q) = \max(0, 1-q)$; linear cost: $c_i(q_i) = c\, q_i$:

$\rightarrow r_2 (q_1) =1/2\ (1- q_1 –c); \quad r_1 (q_2) =1/2\ (1- q_2 –c);$

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- The intersections of the functions r_2 and r_1 are the NE of the Cournot game.
- Example: Linear demand $p(q) = \max(0, 1-q)$; linear cost: $c_i(q_i) = c \, q_i$:

→ $r_2(q_1) = 1/2 \, (1 - q_1 - c)$; $r_1(q_2) = 1/2 \, (1 - q_2 - c)$;

→ NE: $q^*_2 = r_2(q^*_1) = 1/3 \, (1-c) = q^*_1 = r_1(q^*_2)$

Games in Strategic Form & Nash Equilibrium

Nash Equilibrium: Non-Existence-of Pure NE-Example

- Some games may have more than one pure strategy NE
- Not all games have a pure strategy NE:
- Example: Matching pennies:
- Both players simultaneously announce Head or Tails: IF match → 1 wins; If differ → 2 wins
- No pure NE; but mixed strategy NE: ((1/2, 1/2); (1/2, 1/2)) :
- Reasoning: If player 2 plays (1/2, 1/2) then player 1's expected payoff is ½ *1 + ½ *(-1) = 0 when playing H and ½ *(-1) + ½ *1 = 0 when playing T → player 1 is also indifferent

	H	T
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T	-1, 1	1, -1

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Games in Strategic Form & Nash Equilibrium

Nash Equilibrium: Non-Existence--of Pure NE-Example 2

- Another example: Inspection game
- Worker: work or shirk; Employer: Inspect or not inspect
- Worker: working costs g, produces value v; gets wage w
- Employer: Inspection costs h
- We assume $w > g > h > 0$
- If not inspect → worker shirks → better inspect → if inspect → worker always works → better not inspect → ...: No pure NE
- → Employer must randomize

	I	NI
S	0,-h	w,-w
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Games in Strategic Form & Nash Equilibrium

Nash Equilibrium: More than one NE

- Another example: Battle of the sexes
- Man & Woman; Ballet or Football

	B	F
F	0, 0	2, 1
B	1, 2	0, 0

- Another example: Game of chicken
- Driver 1 & Driver 2; Tough or Weak

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→ Let σ_1(F)=x and σ_2(B)=y →
Player 1's indifference:
0 y + 2(1-y) = 1 y + 0 (1-y) → y=2/3
Player 2's indifference:
0 x + 2(1-x) = 1 x + 0 (1-x) → x=2/3
→ Mixed NE: ((2/3, 1/3); (2/3, 1/3))

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- Another example: Game of chicken
- (same reasoning) →

Mixed NE: ((1/2, 1/2); (1/2, 1/2))

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Games in Strategic Form & Nash Equilibrium

Nash Equilibrium: More than one NE

Focal points

- Some games have more than one NE → which will be chosen?
- Theory of „focalness“ of NE („focal points“):
Example: Chose time of day simultaneously; reward if match: 12 noon is focal, 15:37 is not

Risk Dominance

- Stag Hunt: NE: (C;C) and (D;D); (C;C) is pareto-dominant → (C;C) might be chosen if p(C)>0.5 BUT
- more than two players: ALL have to agree on C → $p(C)^8>0.5$ → p(C)>0.93 → (D;D) „risk dominates“ (C;C)

	Hunt Stag (C)	Hunt Hare (D)
Hunt Stag (C)	2 , 2	0 , 1
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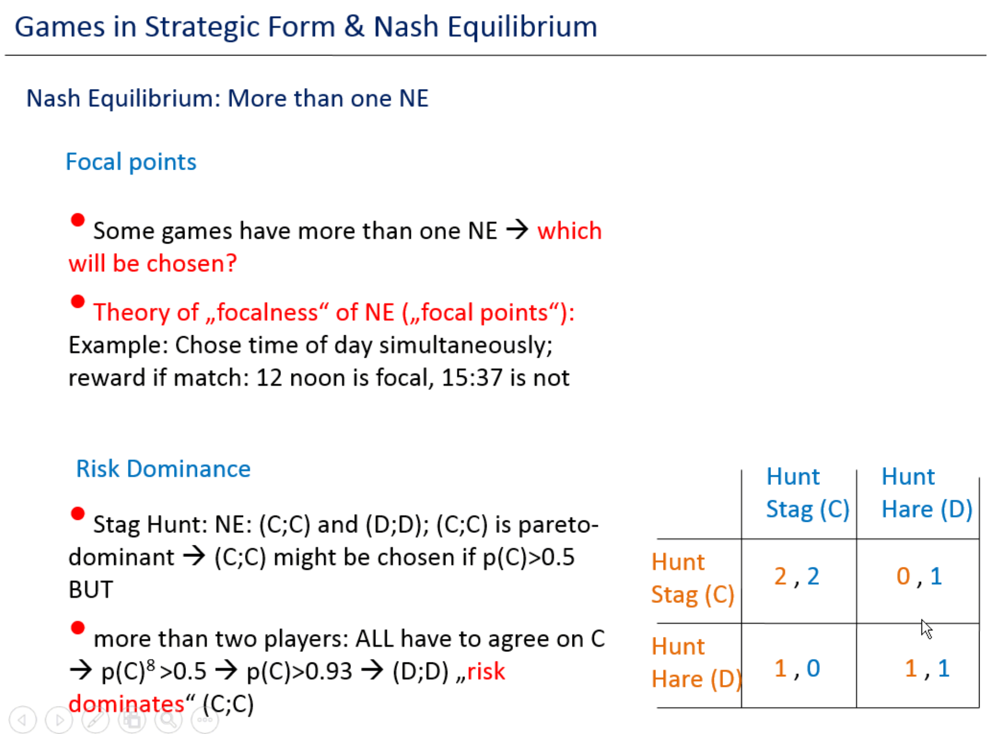

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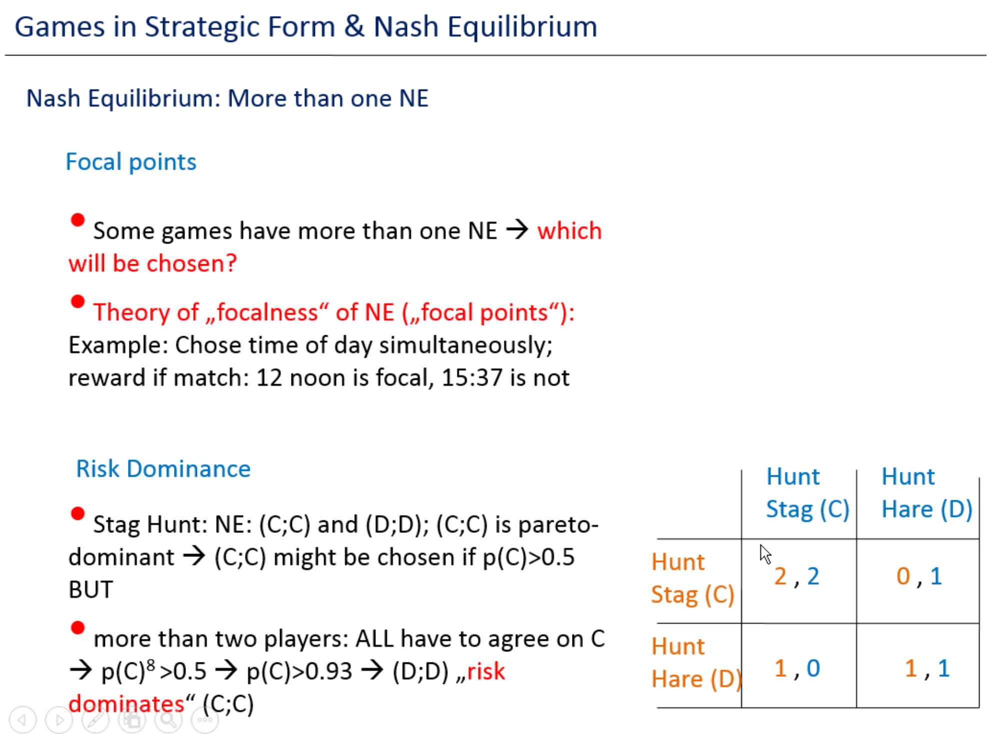

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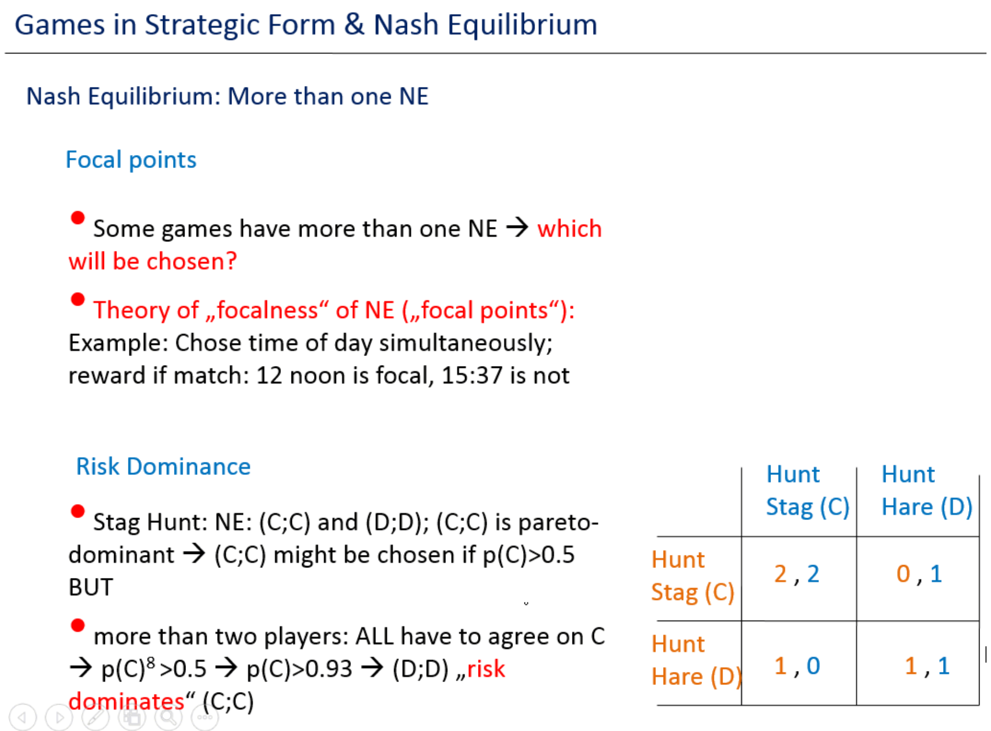

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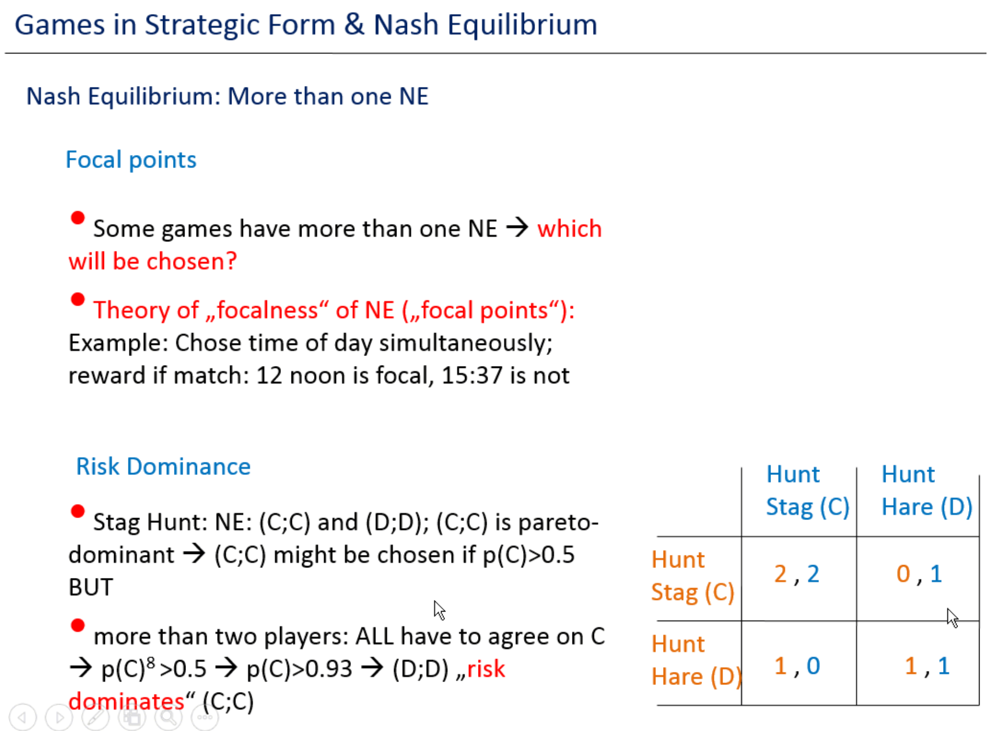

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Games in Strategic Form & Nash Equilibrium

Nash Equilibrium: More than one NE

Risk Dominance / Pareto Optimality

	L	R
U	0,0,10	-5,-5,0
D	-5,-5,0	1,1,-5

A

	L	R
U	-2,-2,0	-5,-5,0
D	-5,-5,0	-1,-1,5

B

- Three player game: Two pure NE: (U,L,A) and (D,R,B); (and one mixed) ; (U,L,A) pareto-dominates (D,R,B)
- If player 3´s choice is fixed→ Two player game → (D,R) is pareto-dominant → if players 1 and 2 expect A : coordinate on (D,R).
- → concept of „coalition proof eq.“ (here (D,R,B))(see [1])

Games in Strategic Form & Nash Equilibrium

Mixed Nash Equilibrium: General Analysis for 2 x 2 Games (see [2])

			Player B	
			q	1-q
			L	R
	p	U	a_{UL}, b_{UL}	a_{UR}, b_{UR}
Player A	1-p	D	a_{DL}, b_{DL}	a_{DR}, b_{DR}

- Pure NE: One cell →
For A: cell's payoff for A must be (weak) maximum over rows in that column
For B: cell's payoff for B must be (weak) maximum over column in that row
- Example: (U,R) is pure NE if $a_{UR} \geq a_{DR}$ and $b_{UR} \geq b_{UL}$